EXAM 3 September 27, 2010

This exam consists of <u>five</u> exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 Consider the propositional formula $\varphi = \neg(p \wedge (q \rightarrow r)) \vee (q \wedge \neg r)$.

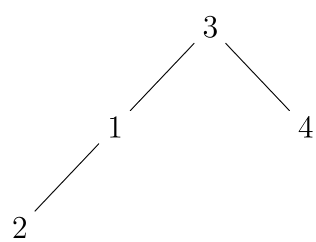
[6] (a) Compute the DAG representation of $T(\varphi)$.

[7] (b) Test the satisfiability of φ with the linear SAT solver.

[7] (c) Transform φ into an equisatisfiable formula in CNF.

2 Consider binary trees where `empty` represents the empty tree and `node(L,E,R)` represents the binary tree with root element `E` and left and right subtrees `L` and `R`.

For example, the tree



is represented by the term

```
node(node(node(empty,2,empty),1,empty),3,node(empty,4,empty)).
```

All of the following exercises can be done independently!

[5] (a) Write a Prolog predicate `height/2` to compute the height of a tree. The height of the example tree is 3.

[5] (b) Write a Prolog predicate `inorder/2` to convert a tree in a list by an inorder-traversal, i.e., first the left subtree, then the element, then the right subtree. For example, `inorder(...,L)` where `...` refers to the example tree should result in `L = [2,1,3,4]`.

[5] (c) Write a Prolog predicate `avl/1` to check whether a given tree is an AVL-tree, i.e., a tree where for every subtree the difference of the heights of the left and right subtree is at most 1. The example tree is an AVL-tree, but if one would remove the 4, then it would not be an AVL-tree.

[5] (d) Write a Prolog predicate `sorted/1` to check whether a given tree is sorted, i.e., where for all nodes with element x, all elements in the left subtree are smaller than x and all elements in the right subtree are larger than x. The example tree is not sorted, but if one would swap the elements 1 and 2, then it would be sorted.

Hint: the predicate `inorder/2` of part (b) might be useful.

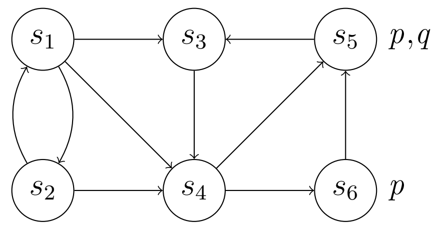
3 For each of the following formulas of predicate logic, either give a natural deduction proof or find a model which does not satisfy it:

[6] (a) $\phi_1 = \forall x\,(A(x) \to B(x)) \to (\exists x\,A(x) \to \exists x\,B(x))$

[7] (b) $\phi_2 = \forall x\,\exists y\,S(x,y) \land \forall x\,\forall y\,(S(x,y) \land S(y,x) \to x = y) \to \forall x\,S(x,x)$

[7] (c) $\phi_3 = \forall x\,(A(x) \to x = c) \to \forall x\,\forall y\,(A(x) \land A(y) \to x = y)$

4 Consider the CTL formulas $\phi = \mathsf{EG}(\mathsf{E}[\neg p\,\mathsf{U}\,q] \land \mathsf{AF}\,q)$, $\psi = \mathsf{EG}\,\mathsf{EF}\,p$, $\chi = \mathsf{EF}\,\mathsf{EG}\,p$ and the model $\mathcal{M}$:



[10] (a) Apply the CTL model checking algorithm to determine the states of $\mathcal{M}$ which satisfy ϕ.

[5] (b) Is ψ equivalent to χ? If not, provide a model and a state which distinguishes ψ from χ, and indicate which of the two formulas is satisfied in the model.

[5] (c) Provide a CTL formula ξ that is satisfied only in state s_1 of $\mathcal{M}$.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
$\forall x\,\phi \to \forall x\,\psi \dashv\vdash \forall x\,(\phi \to \psi)$
Every unary boolean function is affine.
Every adequate set of temporal CTL connectives contains EG or AU.
The terms $p(X,Y,Z)$ and $p(g(Z),g(Y),g(a))$ are unifiable.
Executing the Prolog query `?- [A,B|C] = [A|C].` returns a solution.
The algebraic normal form of the boolean function $f(x,y) = x + \overline{y}$ is $1 \oplus x \oplus xy$.
The term $f(x,y)$ is free for y in $\forall x\,((\forall y\,(P(z) \land \forall z\,Q(y))) \to \neg\forall z\,P(y) \lor Q(z))$.
If θ is a computed answer substitution of an SLD-refutation of P and $\leftarrow A_1,\dots,A_k$ (with $k \geqslant 1$) then $P \vDash (A_1 \lor \cdots \lor A_k)\theta$.
The instance $\{(01,1),(011,0),(0,10),(0,00)\}$ of Post correspondence problem has a solution.
$\mathrm{HWB}_4(0,1,0,1) = 1$