EXAM 1 February 3, 2011

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 Consider the boolean function $f(x,y,z) = 1 \oplus y \oplus xy \oplus xyz$.

[9] (a) Give a binary decision tree for f with the variable ordering $[x,y,z]$ and use the reduce algorithm to construct an equivalent reduced OBDD.

[3] (b) Show that $\neg$ can be expressed in terms of f.

[8] (c) Is $\{f\}$ adequate?

[8] **2** (a) Write a *logic program* which defines the predicate `eval/2`. It should be used to evaluate Boolean expressions which are built from the symbols `and/2`, `not/1`, `true/0`, and `false/0`. For example, the query `eval(and(true,not(false)),X)` should yield the answer `X = true`.

Your program should not evaluate the same expression twice and it should evaluate the second argument of `and/2` only on-demand. To be more precise, if the first argument of `and/2` evaluates to `false`, then the second argument should not be evaluated.

[4] (b) Write another evaluator for Boolean expressions by defining a *Prolog program* for `eval/1`. Here, `eval(E)` should be provable if and only if `E` evaluates to `true`.

In this exercise you must not use `eval/2` from the previous exercise, and you must not define any other predicates besides `eval/1`.

[8] (c) Compute the SLD tree and give all answer substitutions for the following Prolog program and the query `count([p(X,0),p(X,X)],N)`.

```
count([],0).
count([Q|Qs],N)    :- Q, !, count(Qs,N).
count([_|Qs],s(N)) :- count(Qs,N).
p(s(X),X).
```

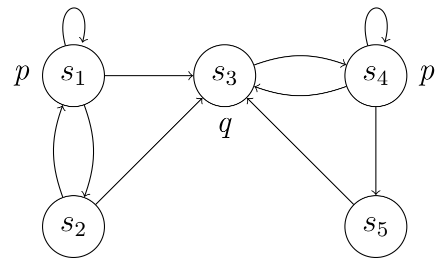
3 For each of the following sequents prove if they are valid or not.

[7] (a) $\vdash \forall x\, ((P(a,x) \wedge P(b,x)) \rightarrow a = b)$

[6] (b) $\vdash \forall x\, (a = b \rightarrow (P(a,x) \rightarrow P(b,x)))$

[7] (c) $\forall x\, P(a,x), a = b \vdash \neg\exists x\, \neg P(b,x)$

4 Consider the CTL formula $\phi = \mathsf{A}[\neg q \,\mathsf{U}(\mathsf{EX}\, p \rightarrow \mathsf{AF}\, q)]$, and the model $\mathcal{M}$:



[10] (a) Apply the CTL model checking algorithm to determine the states of $\mathcal{M}$ which satisfy ϕ.

[5] (b) Provide a CTL formula ψ which is equivalent to ϕ such that ψ does not contain the CTL connectives AU, EX and AF.

[5] (c) Provide a CTL formula χ that is satisfied only in state s_4 of $\mathcal{M}$.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
The sequent $\forall x\, \exists y\, P(x,y), \forall x\, \forall y\, (P(x,y) \rightarrow Q(x,y)) \vdash \exists y\, \forall x\, Q(x,y)$ is valid.
Every affine binary boolean function is self-dual.
In Prolog `xfy` is used for specifying a left-associative infix operator.
Every adequate set of temporal CTL connectives contains EF or AU.
The clauses $\{\neg p, q\}$ and $\{\neg p, r, \neg r\}$ clash.
The terms $p(X, Y, Z)$ and $p(g(Y), g(Z), X)$ are unifiable.
The algebraic normal form of the boolean function $f(x, y) = x \cdot \overline{y}$ is $y \oplus xy$.
$\mathsf{A}[\phi \,\mathsf{U}\, \psi] \equiv \neg(\mathsf{E}[\neg\psi \,\mathsf{U}(\neg\phi \vee \neg\psi)] \wedge \mathsf{EG}\, \neg\psi)$
The set $[\![\, \mathsf{E}[\phi \,\mathsf{U}\, \psi] \,]\!]$ is the least fixed point of F_{EU}.
Prolog programs are executed using SLD resolution with rightmost and topdown selection.