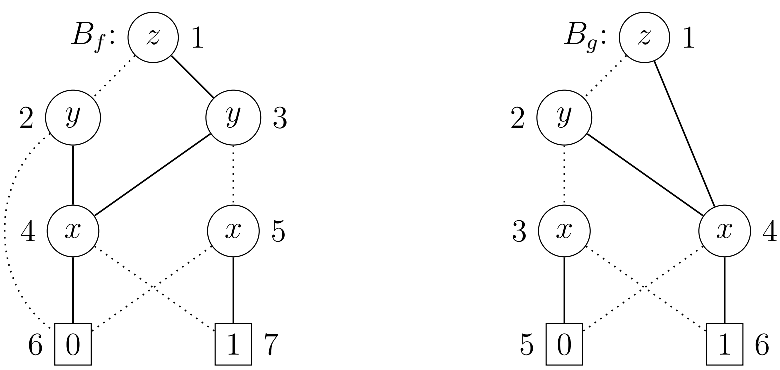
Logik	WS 2011/2012	LVA 703026

EXAM 1 February 2, 2012

This exam consists of <u>five</u> exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 Consider the two OBDDs



[6] (a) Use Shannon's expansion to obtain a boolean expression that is equivalent to B_f.

[7] (b) Compute $\mathsf{apply}(+, B_f, B_g)$.

[7] (c) Compute the algebraic normal form of the boolean function g denoted by B_g.

[7] **2** (a) Compute an equisatisfiable Skolem normal form of

$$\phi = \forall y \, ((\forall x \, P(x) \rightarrow Q(y)) \rightarrow (\exists x \, Q(x) \vee P(y)))$$

[6] (b) Are the terms $s = g(h(x, f(x)), f(z), y)$ and $t = g(h(a, z), f(y), f(a))$ unifiable?

[7] (c) Use resolution to determine satisfiability of the clausal form

$$\{\{P(f(x)), R(x, y)\}, \{\neg Q(x), \neg Q(a)\}, \{\neg R(y, f(z))\}, \{\neg P(f(b)), Q(a)\}\}$$

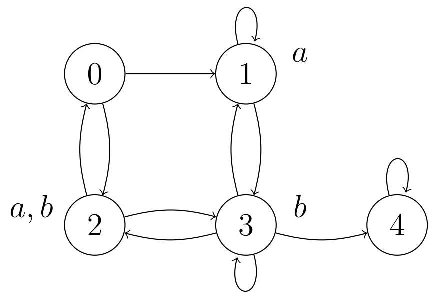
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $p \vee q \vdash \neg p \rightarrow q$

[7] (b) $\forall x \, (P(x) \rightarrow Q(x)), P(b), \neg Q(a), b = a \vdash Q(b)$

[7] (c) $\exists x \, (P(x) \rightarrow Q(x)), P(b), \neg Q(a), b = a \vdash Q(b)$

4 Consider the model $\mathcal{M}$:



[10] (a) Determine in which states of $\mathcal{M}$ the CTL formula $\phi = \mathsf{E}[\mathsf{EX}(a \to b) \,\mathsf{U}\, \mathsf{AG}\,\neg b]$ holds.

[4] (b) Provide a CTL formula ψ that is satisfied only in state 3 of $\mathcal{M}$.

[6] (c) Give a boolean function that represents the transition relation of $\mathcal{M}$.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
Presburger arithmetic is decidable.
Reachability is not expressible in propositional logic.
The backtrack rule of DPLL can simulate the backjump rule of DPLL.
The LTL formulas $\phi \,\mathsf{R}\, \psi$ and $\psi \,\mathsf{W}\, (\phi \vee \psi)$ are semantically equivalent.
A Horn clause $P_1 \wedge P_2 \to Q$ is valid if and only if $Q \in \{P_1, P_2\}$.
$\exists x\, (\exists x\, \psi \to \phi) \dashv\vdash \exists x\, \psi \to \forall x\, \phi$
The algebraic normal form of the boolean function $f(x, y) = x + \overline{y}$ is $1 \oplus x \oplus y$.
The CTL formulas $\mathsf{EG}\,\phi$ and $\neg\mathsf{A}[\top \,\mathsf{U}\, \neg\phi]$ are semantically equivalent.
Every Skolem normal form is in prenex normal form.
To determine whether a network with 3 wires is a sorting network, it suffices to test 6 input sequences.