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| Logik | WS 2011/2012 | LVA 703026 |

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| EXAM 2 | April 13, 2012 |

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

**1** Consider the boolean function $f(x, y, z) = 1 \oplus xy \oplus yz \oplus xyz$.

[8] (a) Give a binary decision tree for $f$ with the variable ordering $[x, y, z]$ and use the reduce algorithm to construct an equivalent reduced OBDD.

[4] (b) Show that the constant function $z(x) = 0$ can be expressed in terms of $f$.

[8] (c) Determine all minimal adequate subsets of $\{\wedge, \vee, \rightarrow, f\}$.

[12] **2** (a) Use first-order resolution to determine the validity of the predicate logic formula

$$\phi = \forall x\, (\neg P(x) \rightarrow P(f(x))) \rightarrow (P(a) \rightarrow \exists x\, (P(x) \wedge P(f(f(x)))))$$

[4] (b) Solve the following instance of Post's correspondence problem given as a sequence of pairs $(s_i, t_i)$:

$$(11, 1), (1, 111), (0111, 10), (10, 0).$$

[4] (c) Are the terms $s = h(g(f(a)), f(g(y)), f(x))$ and $t = h(g(x), f(g(y)), y)$ unifiable?

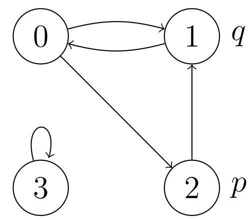
**3** For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $\vdash \forall x\, \exists y\, P(x, y) \rightarrow \exists y\, \forall x\, P(x, y)$

[7] (b) $\vdash \exists y\, \forall x\, P(x, y) \rightarrow \forall x\, \exists y\, P(x, y)$

[7] (c) $\neg(\neg p \wedge \neg q) \vdash p \vee q$

4 Consider the model $\mathcal{M}$:




[8] (a) Determine in which states of $\mathcal{M}$ the CTL formula $\phi = \mathsf{A}[\mathsf{EX}\, p \,\mathsf{U}\, \mathsf{EX}\, q]$ holds.

[8] (b) Determine in which states of $\mathcal{M}$ the LTL formula $\psi = \mathsf{X}p \,\mathsf{U}\, \mathsf{X}q$ holds.

[4] (c) Give a CTL formula $\phi$ such that $\mathcal{M}, s \models \mathsf{X}p \,\mathsf{U}\, \mathsf{X}q$ implies $\mathcal{M}, s \models \mathsf{AF}\phi$ for all models $\mathcal{M}$ and states $s$.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

| statement |
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| Peano arithmetic is decidable. |
| Double negation elimination is a valid proof rule of natural deduction. |
| Restarts are part of basic DPLL. |
| The LTL formulas $\phi \,\mathsf{W}\, \neg\phi$ and $\phi \vee \mathsf{X}\top$ are semantically equivalent. |
| A Horn clause $P_1 \wedge P_2 \rightarrow Q$ is satisfiable if and only if $Q \in \{P_1, P_2\}$. |
| The unification problem $\{x \stackrel{?}{=} y, y \stackrel{?}{=} z\}$ is in solved form. |
| The boolean functions $f(x,y,z) = x \oplus yz \oplus xy$ and $g(x,y,z) = y \oplus xz \oplus xy$ have the same algebraic normal form. |
| The CTL formulas $\mathsf{A}[\phi \,\mathsf{U}\, \neg\phi]$ and $\neg\, \mathsf{EG}\, \phi$ are semantically equivalent. |
| Every prenex normal form is in Skolem normal form. |
| There exists a sorting network with 16 wires and 60 comparators. |