Logik	WS 2011/2012	LVA 703026
EXAM 3		July 11, 2012

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 Consider the propositional formula $\varphi = \neg(\neg(p \wedge \neg q) \rightarrow (\neg q \wedge \neg r))$.

[6] (a) Compute the DAG representation of $T(\varphi)$.

[7] (b) Test the satisfiability of φ with the linear SAT solver.

[7] (c) Transform φ into an equisatisfiable formula in CNF.

[6] 2 (a) Are the terms $s = f(g(x,y), h(a), h(x))$ and $t = f(g(h(a), h(a)), x, h(y))$ unifiable?

[7] (b) Compute a Skolem normal form for

$$\phi = P(a) \wedge \forall x\, (P(x) \wedge \neg \forall x\, (Q(x) \rightarrow \exists x\, P(x)))$$

[7] (c) Use resolution to decide if the following set of clauses is satisfiable:

$$\{\{\neg P(x), Q(x)\}, \{\neg Q(f(a))\}, \{P(f(f(a)))\}, \{P(x), R(f(x))\}, \{\neg Q(x), \neg R(x)\}\}$$

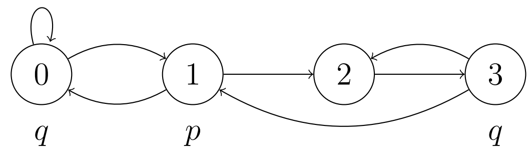
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $(p \rightarrow q) \wedge (r \rightarrow q) \vdash \neg q \rightarrow \neg(p \vee r)$

[7] (b) $\exists y\, P(y) \rightarrow \forall x\, Q(x) \vdash \exists x\, \forall y\, (P(x) \rightarrow Q(y))$

[7] (c) $\forall x\, P(x) \wedge \exists x\, \neg P(x) \vdash \neg(\forall x\, P(x) \wedge \exists x\, \neg P(x))$

4 Consider the model $\mathcal{M}$:



[7] (a) Determine in which states of $\mathcal{M}$ the CTL formula $\phi = \mathsf{E}[\mathsf{AX}\, q \vee p\, \mathsf{U}\, \mathsf{EG}\, q]$ holds.

[7] (b) Give an LTL formula ψ such that $\mathcal{M}, s \models \psi$ if and only if $s = 3$.

[6] (c) Show that the CTL* formulas $\neg\mathsf{E}[\mathsf{G}\,\mathsf{F}\,\neg p]$ and $\mathsf{A}[\mathsf{F}\,\mathsf{A}[\mathsf{G}\, p]]$ are not equivalent.

[20] **5** Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
Every CTL formula can be viewed as a CTL* state formula.
$[\![\, \mathsf{AG}\, \phi \,]\!] = [\![\, \phi \,]\!] \cap \mathsf{pre}_{\forall}([\![\, \mathsf{AG}\, \phi \,]\!])$
The LTL formulas $\phi\, \mathsf{W} \top$ and $\phi\, \mathsf{R} \top$ are semantically equivalent.
Peano arithmetic is decidable.
Reachability is not expressible in second-order logic.
Every complete first-order theory is consistent.
Every adequate set of temporal CTL connectives contains EF or EX or EU.
$\exists x\, \forall y\, (\phi \rightarrow \neg\phi) \dashv\vdash \exists y\, \forall x\, (\neg\psi)$
The boolean function $f(x, y, z) = x \oplus yz$ is affine.
The CTL* formulas $\mathsf{A}[\mathsf{G}\,\mathsf{A}[\mathsf{F}\, p]]$ and $\mathsf{A}[\mathsf{G}\,\mathsf{E}[\mathsf{F}\, p]]$ are semantically equivalent.