Logik WS 2012/2013 LVA 703026

EXAM 1 January 31, 2013

This exam consists of <u>five</u> exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

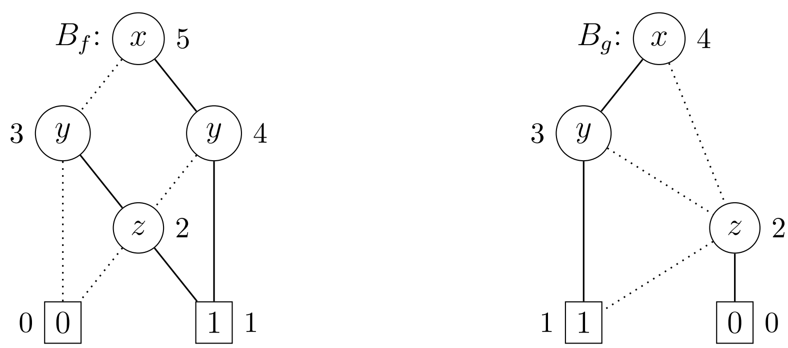
1 Consider the boolean function $f(x, y, z) = x(y + z) \oplus (y \to \bar{x})$.

[6] (a) Give a precise formulation of Post's adequacy theorem.

[7] (b) Compute the algebraic normal form of f.

[7] (c) Determine all minimal complete adequate subsets of $\{\oplus, +, f\}$.

2 Consider the OBDDs



[7] (a) Use Shannon's expansion to obtain a boolean expression that is equivalent to B_g. Simplify your answer.

[6] (b) Compute reduced OBDDs for $f[0/x]$, $f[0/y]$, $f[0/z]$, where f is given by B_f.

[7] (c) Compute $\mathsf{apply}(\oplus, B_f, B_g)$.

3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $(p \wedge q) \vee (r \wedge p) \vdash \neg(s \wedge \neg p)$

[7] (b) $\forall x\,(x = a \vee x = b) \vdash \exists x\,(\neg(x = a) \vee \neg(x = b))$

[7] (c) $\forall x\,(P(x) \to \exists y\, Q(y)),\ P(a) \vdash Q(a)$

[5] $\boxed{4}$ (a) For all $n \in \mathbb{N}$ give a sentence ϕ_n such that $\mathcal{M} \models \phi_n$ if and only if the universe of $\mathcal{M}$ has at least n elements.

[7] (b) Show that finiteness is not expressible in first-order predicate logic. That is, show that there is no sentence ϕ_{fin} such that $\mathcal{M} \models \phi_{\mathsf{fin}}$ if and only if the universe of $\mathcal{M}$ is finite. Hint: Use part (a) and the compactness theorem.

[8] (c) Show that finiteness is expressible in second-order logic. You might want to use the following characterization of infiniteness: There is a transitive and irreflexive binary relation in which every element has a successor.

[20] $\boxed{5}$ Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
The LTL formulas $\phi \,\mathsf{U}\, \psi$ and $\neg(\neg\psi \,\mathsf{U}\, (\neg\psi \wedge \neg\phi)) \wedge \mathsf{F}\psi$ are semantically equivalent.
Every consistent first-order theory is complete.
For every CTL formula ϕ the set $[\![\, \mathsf{AF}\, \phi \,]\!]$ is a fixed point of $F(X) = [\![\, \phi \,]\!] \cap \mathsf{pre}_{\forall}(X)$.
Binary resolution is sound for predicate logic.
The set $\{\mathsf{AX}, \mathsf{EX}, \mathsf{AU}, \mathsf{EU}\}$ is an adequate set of temporal CTL connectives.
The algebraic normal form of the boolean function $f(x, y) = x\overline{y}$ is $y \oplus xy$.
A Horn formula is a disjunction of Horn clauses.
The formula $(p \to q) \wedge (p \to \neg q) \to (p \to r)$ is a theorem of propositional logic.
The predicate logic formulas $\forall x\, \exists y\, P(x, g(y))$ and $\forall y\, P(y, g(h(y)))$ are equisatisfiable.
An LTL formula ϕ is satisfied in a state s of a model $\mathcal{M}$ if there exists a path $\pi\colon s \to \cdots$ in $\mathcal{M}$ that satisfies ϕ.