Logik	WS 2012/2013	LVA 703026
EXAM 2		March 1, 2013

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 Consider the propositional formula $\phi = ((p \vee q) \wedge (\neg p \vee \neg r)) \wedge (\neg p \vee r)$.

[6] (a) Test the satisfiability of ϕ using DPLL.

[7] (b) Test the satisfiability of ϕ using the linear SAT solver.

[7] (c) Test the satisfiability of ϕ using the cubic SAT solver.

[6] **2** (a) Are the terms $s = f(g(x,y), h(a), h(x))$ and $t = f(g(h(a), h(h(a))), x, h(y))$ unifiable?

[7] (b) Compute a Skolem normal form for

$$P(a) \wedge \exists x\, (P(x) \wedge \neg\forall x\, (Q(x) \rightarrow \forall y\, P(y)))$$

[7] (c) Use resolution to decide if the following set of clauses is satisfiable:

$$\{\{Q(a)\}, \{P(x), \neg Q(x)\}, \{Q(f(f(a))), \neg Q(a)\}, \{\neg Q(f(a))\}\}$$

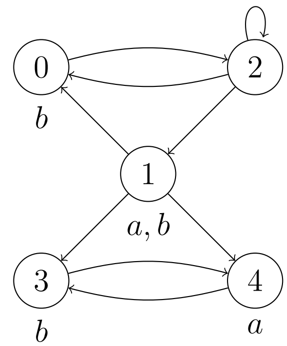
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[7] (a) $\forall x\, (P(x) \rightarrow P(f(x))),\ \forall x\, (P(x) \rightarrow x = a),\ \exists x\, P(x)\ \vdash\ f(a) = a$

[7] (b) $\exists x\, (P(x) \rightarrow P(f(x))),\ \forall x\, (P(x) \rightarrow x = a),\ \exists x\, P(x)\ \vdash\ f(a) = a$

[6] (c) $\vdash\ ((p \rightarrow q) \rightarrow p) \rightarrow p$

4 Consider the model $\mathcal{M}$:



[8] (a) Determine in which states of $\mathcal{M}$ the CTL formula $\phi = \mathsf{AX}\,\mathsf{E}[a\,\mathsf{U}\,b \rightarrow a]$ holds.

[6] (b) Give a CTL formula which holds only in states 2 and 3.

[6] (c) Are the CTL formulas $\mathsf{AF}((\mathsf{AG}\,p) \vee q)$ and $\mathsf{AF}\,\mathsf{AG}(p \vee q)$ equivalent? Prove your answer.

[20] **5** Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
The propositional formula $(p \vee q) \wedge (\neg p \vee \neg q)$ is satisfiable.
$\mathrm{HWB}_5(0,1,0,1,0) = 1$
The set $\{\mathsf{X}, \mathsf{F}, \mathsf{G}\}$ is an adequate set of temporal LTL connectives.
The propositional formula $\top \vee P \vee \bot \rightarrow Q$ is a Horn clause.
The CTL* formulas $\mathsf{EG}\,\mathsf{F}\,p$ and $\mathsf{EG}\,\mathsf{EF}\,p$ are semantically equivalent.
The term $f(x,y)$ is free for y in $\exists x\,(P(x,y) \vee \forall y\,Q(x, f(y,z)))$.
Every sorting network with n wires has at least n comparators.
Peano arithmetic is consistent.
The algebraic normal form of the boolean function $f(x,y) = x\overline{y} \oplus (x+y)$ is y.
For every CTL formula ϕ the set $[\![\,\mathsf{EF}\,\phi\,]\!]$ is a greatest fixed point of $F(X) = [\![\,\phi\,]\!] \cup \mathsf{pre}_{\exists}(X)$.