Logik | WS 2012/2013 | LVA 703026

EXAM 3 | September 25, 2013

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

[5] **1** (a) Are the terms $s = f(g(x,a), h(y,b))$ and $t = f(g(c,z), z)$ unifiable? Here a and b are constants and x, y and z are variables.

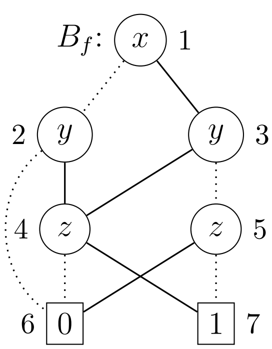
[7] (b) Compute a Skolem normal form for the formula $\forall x\, \exists y\, P(x,y) \wedge \forall x\, Q(x)$.

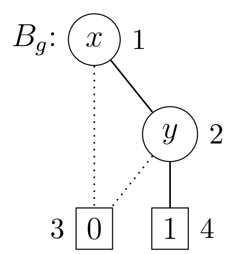
[8] (c) Use resolution to decide if the following set of clauses is satisfiable:

$$\{\{P(a, f(b))\}, \{\neg R(c, f(b))\}, \{P(x,y), R(x,y)\}, \{\neg P(x,y), \neg R(x,y)\},$$
$$\{P(x,y), \neg P(y,x)\}, \{\neg P(x,y), \neg P(y,z), P(x,z)\}, \{R(a,c)\}\}$$

Here a, b and c are constants and x, y and z are variables.

2 Consider the following two OBDDs.





[6] (a) Compute a reduced OBDD for $\forall y.g$, where g is given by B_g.

[7] (b) Use Shannon's expansion to obtain a boolean expression that is equivalent to B_f. Simplify your answer.

[7] (c) Compute $\mathsf{apply}(+, B_f, B_g)$.

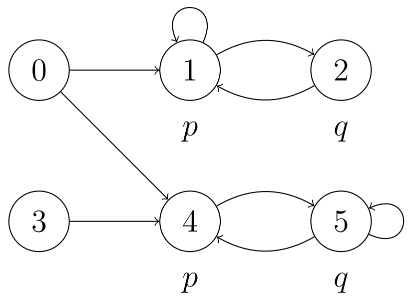
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $p \vdash ((p \to q) \to p) \to p$

[7] (b) $P(a),\ \exists x\,(x = a \to \neg P(x)) \vdash \exists x\, \neg(x = a)$

[7] (c) $Q(a),\ \forall x\,(Q(x) \to Q(f(f(x)))) \vdash \neg Q(f(a)) \vee \forall x\, Q(x)$

4 Consider the model $\mathcal{M}$:



[7] (a) Determine in which states of $\mathcal{M}$ the CTL formula $\phi = \mathsf{EX}\,\mathsf{AF}\,\mathsf{A}[p\,\mathsf{U}\,q]$ holds.

[7] (b) Give an LTL formula ψ such that $\mathcal{M}, s \models \psi$ if and only if $s = 5$.

[6] (c) Find a model and identify a state in that model such that the CTL* formula

$$\mathsf{A}[\mathsf{G}\,\mathsf{F}\,p \wedge \mathsf{G}\,\mathsf{E}[\mathsf{F}\,q]] \wedge \neg\mathsf{A}[p \rightarrow \mathsf{F}\,q]$$

holds in that state.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
The depth of any sorting network is larger than its size.
Presburger arithmetic is finitely axiomatizable.
For every boolean function f and variable x, $f \equiv \overline{x} \cdot f[0/x] \oplus x \cdot f[1/x]$.
Natural deduction for propositional logic without $\neg\neg$e is sound.
Basic DPLL contains the pure literal inference rule.
Every binary self-dual boolean function is monotone.
The CTL formulas $\mathsf{EF}\,\phi \wedge \mathsf{EF}\,\psi$ and $\mathsf{EF}(\phi \wedge \psi)$ are semantically equivalent.
$\forall x\, \phi \rightarrow \exists x\, \psi \dashv\vdash \exists x\, (\phi \rightarrow \psi)$
The boolean function $f(x, y, z) = x + xy$ is affine.
Natural deduction for propositional logic without $\neg\neg$i is complete.