Logik	WS 2013/2014	LVA 703026

EXAM 1 February 6, 2014

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

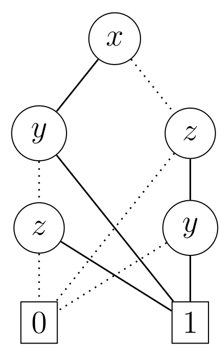
1 Consider the propositional formula $\phi = (\neg p \vee q) \wedge ((\neg q \vee (\neg p \vee r)) \wedge (\neg q \vee \neg r))$.

[7] (a) Test the satisfiability of ϕ using DPLL.

[6] (b) Test the satisfiability of ϕ using the linear SAT solver.

[7] (c) Test the satisfiability of ϕ using the cubic SAT solver.

2 Consider the BDD B_f:



[7] (a) Find a reduced OBDD equivalent to B_f with the ordering $[z, y, x]$.

[6] (b) Use Shannon's expansion to find a simple Boolean expression that is equivalent to B_f.

[7] (c) Apply the `exists` algorithm to B_f and the variable x.

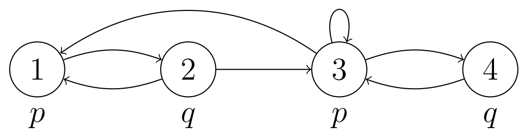
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $\neg(p \vee q \to p \to q \to p) \vdash r$

[7] (b) $\neg\exists x\, R(x,x) \wedge R(a,b) \vdash R(b,a) \to a = b$

[7] (c) $\neg\exists x\, R(x,x),\ R(a,b),\ \forall x\, \forall y\, (R(x,y) \to x = y \vee R(y,x)) \vdash R(b,a)$

[7] 4 (a) Consider the CTL formula $\phi = \mathsf{A}[(p \vee \mathsf{AF}\,\mathsf{EG}\,p)\,\mathsf{U}\,\neg\mathsf{E}[p\,\mathsf{U}(\mathsf{EX}\,\mathsf{AX}\,q)]]$ and the following model $\mathcal{M}$:



Determine which states of $\mathcal{M}$ satisfy ϕ by applying the CTL model checking algorithm.

[6] (b) Show that the CTL* formulas $\neg\mathsf{A}[\mathsf{G}\,\mathsf{F}\,\neg p]$ and $\mathsf{E}[\mathsf{F}\,\mathsf{A}[\mathsf{G}\,p]]$ are not equivalent.

[7] (c) Suppose we extend LTL with a new temporal operator XU:

$$\pi \models \phi\,\mathsf{XU}\,\psi \iff \exists\, i > 1\ \pi^i \models \psi \text{ and } \forall\, 1 < j < i\ \pi^j \models \phi$$

Prove that $\{\mathsf{XU}\}$ is an adequate set of temporal connectives for LTL.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
Natural deduction (including the derived rules) for propositional logic without $\to$e is complete.
Resolution for predicate logic without factoring is sound.
To determine whether a network with n wires is a sorting network, it suffices to test $n!$ input sequences.
The LTL formulas $\phi\,\mathsf{U}\,\psi \vee \mathsf{G}\,\phi$ and $\psi\,\mathsf{R}\,(\phi \vee \psi)$ are semantically equivalent
Presburger arithmetic is decidable.
Every unification problem can either be solved or leads to a failure due to the occurs check.
Tseitin's transformation can be performed in linear time.
There exist formulas in CNF that are not in Skolem normal form.
Every complete first-order theory is consistent.
There exist CTL* formulas without equivalent CTL or LTL formulas.