Logik	WS 2013/2014	LVA 703026

EXAM 2	February 28, 2014

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 Consider the boolean function $f = x(y \oplus z) + (\overline{y} + x)$.

[6] (a) Give a precise formulation of Post's adequacy theorem.

[7] (b) Compute the algebraic normal form of f.

[7] (c) Determine all minimal complete adequate subsets of $\{f, \neg, \rightarrow\}$.

[7] **2** (a) Use the unification algorithm to determine whether the terms $f(f(y, h(x)), f(h(y), z))$ and $f(f(z, w), f(h(x), w))$ are unifiable. Here w, x, y and z are variables.

[6] (b) Compute a Skolem normal form of $\forall x\, (\neg\forall y\, (P(x) \rightarrow (\exists x\, (Q(x) \vee R(y, x))) \wedge R(x, y)))$.

[7] (c) Use resolution to decide whether the following clausal form is satisfiable:

$$\begin{aligned}&\{\{\neg P(f(x), x),\ \neg P(y, a)\},\\ &\ \{\neg R(x, y),\ Q(b, x),\ P(x, y)\},\\ &\ \{R(x, y),\ \neg Q(z, f(y)),\ P(f(a), z)\},\\ &\ \{R(z, y),\ Q(a, z),\ P(z, a)\},\\ &\ \{\neg Q(b, f(x))\}\}\end{aligned}$$

Here a and b are constants, and x, y and z are variables.

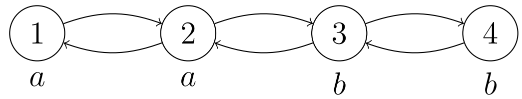
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $p \rightarrow (q \vee (p \rightarrow q)) \vdash q \rightarrow \neg p$

[7] (b) $\exists x\, (P(x) \vee \neg Q(x)) \vdash \exists x\, \neg(Q(x) \wedge \neg P(x))$

[7] (c) $\exists x\, (P(x) \wedge \exists y\, (\neg(x = y) \wedge \neg Q(y))) \vdash \neg\forall x\, (P(x) \wedge Q(x))$

[6] 4 (a) Consider the CTL formula $\phi = \mathsf{E}[(\mathsf{EG}\, a \wedge \mathsf{AX}\,\mathsf{EF}\, b)\, \mathsf{U}\, \mathsf{EX}\, \mathsf{A}[a\, \mathsf{U}\, \mathsf{EG}\, b]]$ and the following model $\mathcal{M}$:



Use the CTL model checking algorithm to determine in which states of $\mathcal{M}$ the formula ϕ is satisfied.

[7] (b) Find a CTL formula that is satisfied only in states 2 and 4 of $\mathcal{M}$.

[7] (c) Show that the LTL formulas $\phi\, \mathsf{W}\, \psi$ and $\phi\, \mathsf{U}\, \psi \vee (\phi \wedge \mathsf{XF}\phi)$ are not semantically equivalent.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
There exists a consistent and incomplete first-order theory.
A path $s_1 \to s_2 \to \cdots$ is fair with respect to a set C of fairness constraints if for all $\psi \in C$ we have $s_j \models \psi$ for at least one $j \geqslant 1$.
There is an efficient decision procedure for satisfiability of Horn formulas.
The proof rules LEM, PBC and ¬¬e are inter-derivable with respect to the other basic proof rules of natural deduction for propositional logic.
Satisfiability of CTL formulas is decidable for finite models.
$\neg\forall x\, P(x) \dashv\vdash \exists y\, \neg P(y)$
A set of sentences of predicate logic is satisfiable if all its finite subsets are satisfiable.
The CTL formulas $\mathsf{A}[\phi\, \mathsf{U}\, \psi]$ and $\neg(\mathsf{E}[\neg\psi\, \mathsf{U} (\neg\phi \wedge \neg\psi)] \vee (\mathsf{EG}\, \psi))$ are semantically equivalent.
The set $[\![\mathsf{AF}\, \phi]\!]$ is the greatest fixed point of the function $F_{\mathsf{AF}}(X) = [\![\phi]\!] \cup \mathsf{pre}_{\forall}(X)$.
If reduced OBDDs B_1 and B_2 represent the same function then B_1 and B_2 have identical structure.