Logik	WS 2013/2014	LVA 703026

EXAM 3 — September 22, 2014

> This exam consists of <u>five</u> exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

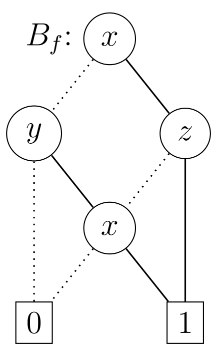
1 Consider the Boolean function $f(x,y,z) = (\overline{y} + z) \oplus y(x+z)$.

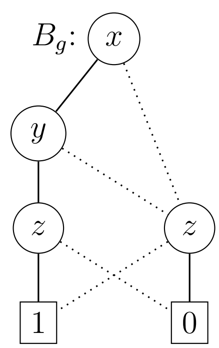
[6] (a) Give precise definitions of the following properties of Boolean functions: monotonicity, self-duality, and affinity.

[7] (b) Compute an algebraic normal form of f.

[7] (c) Determine all minimal complete adequate subsets of $\{f, \bot, \rightarrow\}$.

2 Consider the BDDs





[7] (a) Use Shannon's expansion to obtain a boolean expression equivalent to B_f. Simplify your answer.

[6] (b) Transform B_g into an equivalent reduced OBDDs with variable ordering $[y, z, x]$.

[7] (c) Compute a reduced OBDD for $\forall z.g$, where g is given by B_g.

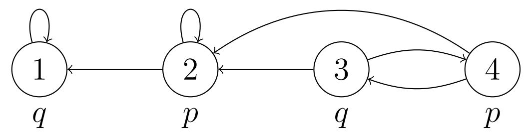
3 Either give a natural deduction proof or find a model which does not satisfy the following sequents. In the natural deduction proofs you may additionally use the quantifier equivalences (e.g. $\neg\forall x\, \phi(x) \dashv\vdash \exists x\, \neg\phi(x)$).

[5] (a) $\forall x\, \forall y\, (x = y \rightarrow P(x,y)),\ \exists x\, \neg(P(x,a) \vee P(a,x)) \vdash \neg(a = b) \rightarrow P(a,b)$

[10] (b) $\vdash \exists x\, (\neg P(x) \rightarrow \forall y\, \neg P(y))$

[5] (c) $\vdash \forall x\, (\neg P(x) \rightarrow \forall y\, \neg P(y))$

4 Consider the following model $\mathcal{M}$:



[7] (a) Determine in which states of $\mathcal{M}$ the CTL formula $\phi = \mathsf{E}[(\mathsf{A}[\mathsf{EF}\, q \,\mathsf{U}\, \mathsf{EG}\, p] \vee \mathsf{AX}\, p) \,\mathsf{U}\, \mathsf{AG}\, q]$ holds.

[7] (b) Give an LTL formula ψ such that $\mathcal{M}, s \models \psi$ if and only if $s = 4$.

[6] (c) Show that the two CTL formulas with fairness constraints $\mathsf{E}_{\{p,q\}}\mathsf{G}\,\top$ and $\mathsf{E}_{\{p \wedge q\}}\mathsf{G}\,\top$ are not equivalent.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
There exists a CTL model $\mathcal{M}$ and a state s such that $\mathcal{M}, s \models \mathsf{E}_{\{\neg\phi\}}\mathsf{G}\,\phi$.
The theorem of Löwenheim and Skolem states that a set of sentences of predicate logic is satisfiable if all its finite subsets are satisfiable.
The quantifier-free fragment of the theory of arrays is decidable.
The terms $f(g(x), x, y)$ and $f(z, y, g(z))$ are unifiable. Here x, y, and z are variables.
The set $\{\mathsf{U}, \mathsf{R}\}$ is an adequate set of connectives for the LTL fragment consisting of negation-normal forms without X.
There exists a sorting network for 16 wires with 60 comparators.
The LTL formulas $\phi \,\mathsf{R}\, \psi$ and $\psi \,\mathsf{W}\, (\psi \wedge \phi)$ are semantically equivalent.
The predicate logic formula $\forall x\, \exists y\, ((P(x) \wedge Q(x, y)) \vee \neg Q(y))$ is in Skolem normal form.
The boolean function $f(x, y, z) = x + xy$ is self-dual.
The use of the backjump rule in abstract DPLL is known as chronological backtracking.