Logik	WS 2014/2015	LVA 703026

EXAM 1	February 5, 2015

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

[7] **1** (a) Prove the validity of the sequent

$$\vdash p \vee \neg p$$

using only the *basic* proof rules of natural deduction.

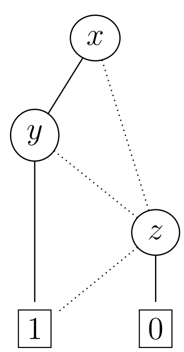
[6] (b) Use DPLL to test the satisfiability of the propositional formula

$$\phi \;=\; (p \vee \neg q \vee r) \wedge (\neg p \vee \neg q) \wedge (p \vee \neg q \vee \neg r) \wedge (q \vee \neg r) \wedge (q \vee r)$$

[7] (c) Compute the algebraic normal form of the boolean function

$$f(x, y, z) \;=\; x + (x + y) \cdot \overline{(y + z)}$$

2 Consider the boolean function $f(x, y, z) = x \oplus xy \oplus yz$ and the BDD B_g:



[7] (a) Construct a reduced OBDD for f with the variable ordering $[x, y, z]$.

[6] (b) Use Shannon's expansion to obtain a boolean function that is equivalent to B_g.

[7] (c) Compute $\mathsf{apply}(\oplus, B_g, B_g)$.

3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $p \to q \to r \;\vdash\; \neg(p \wedge (q \wedge \neg r))$

[7] (b) $Q(a, a) \;\vdash\; \exists y\, \forall x\, (x = y \to Q(y, x))$

[7] (c) $\exists y\, \forall x\, (x = y \to Q(y, x)) \;\vdash\; Q(a, a)$

[7] $\boxed{4}$ (a) Consider the model $\mathcal{M} = (S, \to, L)$ with

$$S = \{1,2,3,4\}$$
$$\to = \{(1,1),(1,2),(1,3),(1,4),(2,3),(3,4),(4,4)\}$$

and L given as $L(1) = \{p\}$, $L(2) = \{p,q\}$, $L(3) = \{q\}$, $L(4) = \varnothing$. Use the labelling algorithm to determine the states in which the formula $\mathsf{E}[\mathsf{EF}(p \wedge q)\,\mathsf{U}(p \vee q)]$ is satisfied.

[6] (b) Consider the two CTL formulas $\phi = \mathsf{AX}\,p$ and $\psi = \mathsf{E}[p\,\mathsf{U}\,q]$.

i. Find a model $\mathcal{M}$ and a state s such that $\mathcal{M}, s \models \phi$ and $\mathcal{M}, s \not\models \psi$.

ii. Find a model $\mathcal{M}$ and a state s such that $\mathcal{M}, s \not\models \phi$ and $\mathcal{M}, s \models \psi$.

[7] (c) Give an LTL formula which is not expressible in CTL.

[20] $\boxed{5}$ Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
Presburger arithmetic is finitely axiomatizable.
$\exists x\,(\phi \wedge \psi) \dashv\vdash \exists x\,\phi \wedge \exists x\,\psi$
Every path formula in CTL* is also a state formula.
The depth of any sorting network for 16 wires is at least 9.
$\mathsf{E}_C\mathsf{X}\,\phi \equiv \mathsf{EX}((\mathsf{E}_C\mathsf{G}\,\top) \wedge \phi)$
Resolution without factoring is unsound for predicate logic.
The conjunction of the negation of all decision literals when a conflict occurs is a backjump clause in basic DPLL.
The LTL formulas $\phi\,\mathsf{U}\,\neg\psi$ and $\neg\mathsf{G}\psi \wedge \neg\psi\,\mathsf{R}\,(\psi \to \phi)$ are semantically equivalent.
There are more affine binary boolean functions than non-affine binary boolean functions.
For every predicate logic formula there exists an equisatisfiable formula in prenex normal form.