EXAM 2 February 27, 2015

This exam consists of <u>five</u> exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

[7] **1** (a) Consider the propositional formula

$$\phi \;=\; \neg(\neg(p \wedge r) \rightarrow (r \wedge q))$$

Compute the DAG representation of $T(\phi)$ and test the satisfiability of ϕ with the linear SAT solver.

[6] (b) Transform the propositional formula

$$(p \wedge q) \vee (\neg r \wedge (p \vee q))$$

into an equisatisfiable CNF by applying Tseitin's transformation.

[7] (c) Use DPLL to test the satisfiability of the propositional formula

$$(q \vee r) \wedge (\neg q \vee \neg p) \wedge (\neg q \vee p)$$

[7] **2** (a) Determine whether the terms $h(g(x,y,y),f(a))$ and $h(g(f(z),x,f(a)),f(z))$ are unifiable and compute a most general unifier, if possible. Here a is a constant and x, y and z are variables.

[13] (b) Use resolution to determine validity of the following formula:

$$\forall x\, \exists y\, P(x,y) \rightarrow \exists y\, \forall x\, P(x,y)$$

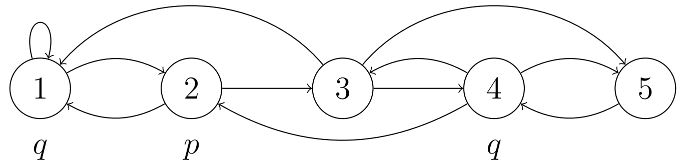
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $p \;\vdash\; q \vee \neg(q \rightarrow p)$

[7] (b) $\forall x\, (P(x) \vee Q(x)), \exists x\, \neg P(x), \forall x\, (R(x) \rightarrow \neg Q(x)) \;\vdash\; \exists x\, \neg R(x)$

[7] (c) $\exists x\, \exists y\, (\neg(x = y) \wedge \forall z\, (x = z \vee z = y)) \;\vdash\; \exists x\, \forall y\, (x = y)$

[6] $\boxed{4}$ (a) Use the labelling algorithm to determine in which states of the model



the CTL formula $\mathsf{AX}\,\mathsf{E}[\neg p\,\mathsf{U}\,\mathsf{EG}\,q]$ holds.

[7] (b) Prove that the CTL* formulas $\mathsf{E}[\mathsf{G}\,\mathsf{F}\,p]$ and $\mathsf{EG}\,\mathsf{EF}\,p$ are not equivalent.

[7] (c) Construct a CTL formula that only holds in states 1 and 3 of the model of part (a).

[20] $\boxed{5}$ Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
For every propositional formula Tseitin's transformation produces an equivalent CNF.
Every propositional formula can be written as a Horn formula.
Satisfiability of CTL formulas in finite models is decidable.
$\forall x\,\exists y\,(\neg Q(x) \wedge Q(y)) \;\equiv\; \exists x\,\forall y\,(\neg Q(x) \wedge Q(y))$
Reachability is expressible in (full-fledged) second-order logic.
To test if a network with n inputs is a sorting network it suffices to consider 2^n input combinations.
Every BDD can be represented as an affine boolean function.
For every LTL formula there is an equivalent CTL formula.
If a predicate logic formula φ has a model with infinitely many elements then it has a model with n elements for all $n \geqslant 1$.
If a boolean function is monotone and self-dual then it is not affine.