Logik	WS 2014/2015	LVA 703026

EXAM 3 — September 25, 2015

> This exam consists of <u>five</u> exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 Consider the boolean function $f(x,y) = (x+y)\cdot\overline{(x\cdot y)}$.

[4] (a) Compute the algebraic normal form of f.

[3] (b) Is $\{f\}$ adequate?

[6] (c) Show that there is no nullary, unary, or binary boolean function that is self-dual but not affine. (Hint: There are only finitely many boolean functions with arity $\leqslant 2$.)

[7] (d) Find a ternary boolean function that is self-dual but not affine.

[6] **2** (a) Determine whether the terms $f(f(x,y),f(x,z))$ and $f(z,f(y,z))$ are unifiable and compute a most general unifier, if possible.

[7] (b) Compute a Skolem normal form for the predicate logic formula

$$P(a) \wedge \neg\forall x\,(P(x) \rightarrow \exists x\,(Q(x) \rightarrow \forall y\,P(y)))$$

[7] (c) Use resolution to decide if the following set of clauses is satisfiable:

$$\{\{P(a)\},\{Q(x),\neg P(f(x))\},\{P(f(f(x))),\neg P(x)\},\{\neg Q(f(a))\}\}$$

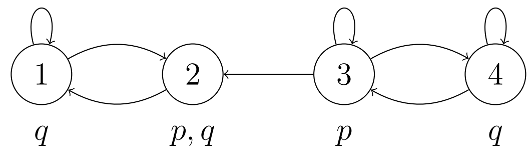
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $q \rightarrow p \;\vdash\; p \rightarrow \neg q \rightarrow p$

[7] (b) $\forall x\,(Q(x) \rightarrow P(x)), \exists x\,\neg P(x) \;\vdash\; \exists x\,\neg Q(x)$

[7] (c) $\forall x\,(Q(x) \rightarrow P(x)), \exists x\,\neg Q(x) \;\vdash\; \exists x\,\neg P(x)$

4 Consider the following model $\mathcal{M}$:



[7] (a) Use the labelling algorithm to determine in which states of the model $\mathcal{M}$ the CTL formula $\mathsf{E}[\mathsf{EX}\, p\, \mathsf{U}\, \mathsf{AX}\, q \wedge \mathsf{EG}\, q]$ holds.

[6] (b) Give a CTL* formula ψ such that $\mathcal{M}, s \models \psi$ if and only if $s = 4$.

[7] (c) Show that the CTL formulas with fairness constraints $\mathsf{E}_{\{p,q\}}\mathsf{G}\top$ and $\mathsf{E}_{\{p \vee q\}}\mathsf{G}\top$ are not equivalent.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
The LTL formulas $\neg(\phi\, \mathsf{R}\, \psi)$ and $\neg\phi\, \mathsf{U}\, \neg\psi$ are semantically equivalent.
The compactness theorem holds for universal second-order logic.
A boolean function is affine if it can be written in algebraic normal form.
Every Horn formula can be written in conjunctive normal form.
We have $\mathcal{M} \models \phi$ for sentences ϕ in predicate logic.
The set of connectives $\{\bot, \vee, \leftrightarrow\}$ is adequate for propositional logic.
There is a boolean function f with n arguments such that every OBDD computing f has size exponential in n, for all natural numbers n.
$s = t, t = u \vdash s = u$
Validity in predicate logic is decidable using resolution with factoring.
Gödel's incompleteness theorem states that there is a sentence that is true in the standard model of arithmetic but not provable in PA.