Logik	WS 2015/2016	LVA 703026

EXAM 1 — February 4, 2016

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 Consider the boolean functions $f(x, y) = \overline{x} \oplus y$ and $g(x, y, z) = (x + y)(\overline{x} + z) + y\overline{z}$.

[6] (a) Construct a reduced OBBD for f with the variable ordering $[x, y]$.

[7] (b) Compute the algebraic normal form of g.

[7] (c) Determine all minimal complete adequate subsets of $\{f, g, \to, \oplus\}$.

[7] **2** (a) Consider the terms $h(x_1, x_2, f(a, b), f(y_1, y_1), y_2)$ and $h(f(x_0, x_0), f(x_1, x_1), y_1, y_2, x_2)$. Determine whether they are unifiable and compute a most general unifier if possible. Here a and b are constants and x_0, x_1, x_2, y_1, and y_2 are variables.

[6] (b) Compute a Skolem normal form of

$$\phi = \exists y\, ((\forall x\, P(x) \to \exists x\, Q(x)) \to \exists x\, \forall y\, R(x, y) \to P(y))$$

[7] (c) Use resolution to determine whether the following set of clauses is satisfiable:

$$\begin{aligned}\{&\{P(f(b), z), Q(x, y), \neg R(z, f(u))\},\\ &\{P(x, x), \neg Q(x, y), R(a, x)\},\\ &\{P(z, a), Q(x, y), R(f(b), y)\},\\ &\{\neg R(a, f(x))\},\\ &\{\neg P(x, y), \neg P(z, z)\}\}\end{aligned}$$

Here a and b are constants and x, y, z, and u are variables.

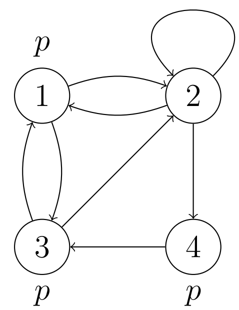
3 For each of the following sequents (the first in propositional logic, the latter two in predicate logic), either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $\vdash\ ((p \to \bot) \to p) \to p$

[7] (b) $\exists x\, \forall y\, (x = y \to Q(y))\ \vdash\ \exists x\, Q(x)$

[7] (c) $\forall x\, (P(x) \to P(f(f(x)))),\ P(a)\ \vdash\ P(f(f(f(a))))$

4 Consider the following model $\mathcal{M}$:



[6] (a) Determine in which states of $\mathcal{M}$ the CTL formula $\phi = \mathsf{A}[\mathsf{EX}\,\neg p \wedge p\,\mathsf{U}\,\mathsf{EG}\,p]$ is satisfied by applying the CTL model checking algorithm.

[7] (b) Find a CTL formula ψ that is satisfied only in states 2 and 4 of $\mathcal{M}$.

[7] (c) Find an LTL formula χ such that neither $\mathcal{M}, 4 \vDash \chi$ nor $\mathcal{M}, 4 \vDash \neg\chi$.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
$\forall x\,(\mathsf{G}x)$ is an LTL formula.
$\exists x\,(P(x) \vee Q(x)) \models \exists x\,P(x) \vee \exists x\,Q(x)$
Unification of two terms is undecidable in general.
The formula $\forall x\,P(x) \wedge Q(x) \to \forall x\,(P(x) \wedge Q(x))$ is valid.
Reflexivity of a binary relation is expressible in predicate logic.
The set of connectives $\{\bot, \neg\}$ is adequate for propositional logic.
A predicate logic formula ϕ is satisfiable if and only if $\neg\phi$ is not valid.
A predicate logic formula can at the same time be satisfiable *and* valid.
The set $\{R(x), S(x) \to \neg R(x)\}$ of predicate logic formulas is consistent.
The universe $A = \{0\}$ together with the relations $P^{\mathcal{M}} = Q^{\mathcal{M}} = \{0\}$ constitutes a countermodel of $\forall x\,(P(x) \vee Q(x)) \models \forall x\,Q(x) \vee \forall x\,P(x)$.