Logik WS 2015/2016 LVA 703026

EXAM 2 March 4, 2016

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 Let S be the clausal form $\{\{\neg p, \neg q\}, \{p, q\}, \{\neg q, \neg r\}, \{q, r\}, \{\neg r, \neg p\}, \{r, p\}\}$.

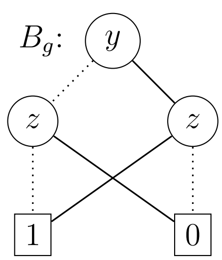
[6] (a) Give the propositional logic formula that is respresented by S, and explain why or why not it is a Horn formula.

[7] (b) Use resolution to decide whether S is satisfiable.

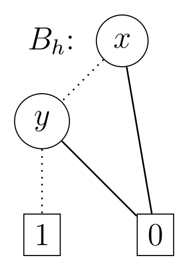
[7] (c) Describe a procedure to check by means of resolution whether or not $\phi_1, \phi_2 \models \psi$, for arbitrary propositional formulas ϕ_1, ϕ_2 and ψ. Subsequently use the procedure to show that (i) $p, p \to q \models q$ and (ii) not $p, q \to p \models q$.

2 Consider the boolean function $f(x, y) = x\overline{y} \oplus \overline{x}y$ and the following two reduced OBDDs:

B_g:



B_h:



[6] (a) Construct a reduced OBDD for f with variable ordering $[x, y, z]$.

[7] (b) Compute $\mathsf{apply}(+, B_g, B_h)$.

[7] (c) Starting from B_g, compute a reduced OBDD that is equivalent to $\exists y.g$.

3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $\forall x\, \exists y\, (\neg(x = y) \land \exists z\, (x = z \to z = y)) \;\vdash\; \exists x\, \forall y\, (x = y)$

[7] (b) $\vdash\; \forall x\, \exists y\, (P(x) \to Q(y)) \to \forall x\, (P(x) \to \exists y\, Q(y))$

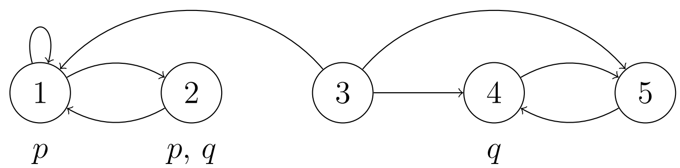
[7] (c) $\exists x\, \neg P(x),\; \forall x\, (P(x) \lor Q(x)) \;\vdash\; \forall x\, (R(x) \to \neg Q(x)) \to \exists x\, \neg R(x)$

4 Let the *strong* until operator S in LTL be defined by $\phi \,\mathsf{S}\, \psi$ if there exists $i \geqslant 1$ such that $\pi^i \models \psi$ and, for all $j < i$, $\pi^j \models \phi \wedge \neg\psi$.

[6] (a) Give a model M such that $\pi_1 \models p \,\mathsf{S}\, q$ but $\pi_2 \not\models p \,\mathsf{S}\, q$ for two paths π_1 and π_2 starting from the same state.

[6] (b) Show that $\{\mathsf{X}, \mathsf{S}\}$ (together with the propositional connectives) is adequate for LTL.

[8] (c) Use the labelling algorithm to determine in which states of the model



the CTL formula $\mathsf{AF}\,\mathsf{E}[\neg p \,\mathsf{U}\, \mathsf{EX}\, q]$ holds.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
$\forall x\, \exists y\, (\neg P(x,y) \vee Q(y)) \;\;\dashv\vdash\;\; \exists x\, \forall y\, (\neg P(x,y) \vee Q(y))$
There is a sorting network of size three for four inputs.
The boolean function $f(x,y,z) = \overline{x} + xy + x\overline{y}$ is affine.
For every LTL formula there is an equivalent CTL* formula.
Every affine boolean function can be written in algebraic normal form.
The term $g(f(z))$ is free for x in $\exists x\, \forall z\, (Q(x) \vee P(z)) \wedge \forall y\, (Q(y) \vee P(x))$.
The set $\{\mathsf{EX}, \mathsf{EG}, \mathsf{EU}\}$ is an adequate set of temporal connectives for CTL.
The formula $((\neg r \to \neg s) \to r) \to p \to s \to p$ is a theorem of propositional logic.
For any monotone function $F\colon \mathcal{P}(S) \to \mathcal{P}(S)$ with $\lvert S \rvert = n$, $F^n(\varnothing)$ is the greatest fixed point of F.
The terms $f(g(x,y), h(a), h(x))$ and $f(g(h(a), h(h(a))), x, h(y))$ are unifiable. Here a is a constant and x and y are variables.