Logik | WS 2015/2016 | LVA 703026

EXAM 3 | September 26, 2016

This exam consists of <u>five</u> exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

[7] **1** (a) Consider the propositional formula $\varphi = \neg(q \vee (p \rightarrow r)) \wedge (p \vee \neg r)$. Use Tseitin's transformation to transform φ into an equisatisfiable CNF.

[6] (b) Use DPLL to test the satisfiability of the propositional formula

$$\phi \;=\; (s \vee q) \wedge (\neg s \vee q) \wedge (t \vee p) \wedge (\neg t \vee \neg q) \wedge (t \vee \neg q \vee \neg p)$$

[7] (c) Consider the propositional formula $\psi = 3 \wedge (\neg 1 \vee \neg 3 \vee 2) \wedge (\neg 2 \vee 4) \wedge (\neg 2 \vee \neg 4)$ and the following (partial) DPLL derivation ending in a conflict:

$$\begin{array}{rrcll}
 & & \| & \psi & \\
\Longrightarrow & 3 & \| & \psi & \text{(unit propagation)} \\
\Longrightarrow & 3\ \overset{d}{1} & \| & \psi & \text{(decide)} \\
\Longrightarrow & 3\ \overset{d}{1}\ 2 & \| & \psi & \text{(unit propagation)} \\
\Longrightarrow & 3\ \overset{d}{1}\ 2\ 4 & \| & \psi & \text{(unit propagation)}
\end{array}$$

Draw the conflict graph, determine all unique implication points with the corresponding backjump clauses, and give the results obtained by applying the backjump rule using those clauses.

[6] **2** (a) Determine whether the terms $f(f(f(f(a,z),y),x),w)$ and $f(w,f(x,f(y,f(z,a))))$ are unifiable and compute a most general unifier, if possible. Here a is a constant symbol and x, y, z, and w are variables.

[7] (b) Compute a Skolem normal form for the predicate logic formula

$$\exists x\, P(x) \rightarrow (\neg\forall x\, (Q(x) \rightarrow \exists x\, R(x)) \rightarrow \forall y\, P(y))$$

[7] (c) Use resolution to determine whether the following set of clauses is satisfiable:

$$\{\{P(a)\}, \{\neg P(x), P(f(x))\}, \{\neg P(f(f(f(a))))\}\}$$

3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $\forall x\, \exists y\, (P(x) \to Q(y)) \ \vdash\ \forall x\, (P(x) \to \exists y\, Q(y))$

[7] (b) $\forall x\, (P(x) \to \exists y\, Q(y)) \ \vdash\ \forall x\, \exists y\, (P(x) \to Q(y))$

[7] (c) $\forall x\, \exists y\, (P(x) \to R(x, y)) \ \vdash\ \exists y\, \forall x\, (P(x) \to R(x, y))$

[7] **4** (a) Complete the following proof of the Löwenheim-Skolem theorem (which states that a sentence ψ has a model with infinitely many elements if it has a model with at least n elements for all $n \geqslant 1$) for first-order logic by filling in the missing parts:

Consider the family of sentences

$$\phi_n = \exists x_1\, \exists x_2\, \ldots\, \exists x_n \bigwedge_{1 \leqslant i < j \leqslant n} \boxed{\qquad\qquad\qquad\qquad}$$

and the infinite set

$$\Gamma = \boxed{\qquad\qquad\qquad\qquad}$$

Every finite subset Δ of Γ is $\boxed{\qquad\qquad}$. To see that, let

$$k = \max \boxed{\qquad\qquad\qquad\qquad}$$

and consider a model with at least k elements satisfying ψ, which exists by assumption. Hence by the $\boxed{\qquad\qquad}$ theorem also Γ is satisfiable, i.e., there is a model $\mathcal{M}$ with $\boxed{\qquad\qquad}$. So $\mathcal{M} \models \phi_n$ for all $n \geqslant 1$ and also $\mathcal{M} \models \boxed{\qquad\qquad}$. Thus $\mathcal{M}$ has infinitely many elements and satisfies ψ.

[7] (b) Determine whether the following formula in second-order logic is valid:

$$\forall P\, (\forall x\, \forall y\, \forall z\, (P(x, y) \wedge P(y, z) \to P(x, z)) \wedge \neg \forall x\, \forall y\, (P(x, y) \to P(y, x)) \to \forall x\, \neg P(x, x))$$

[6] (c) Consider a model $\mathcal{M}$ consisting of a single binary predicate R, that represents a directed graph G. Express the following two statements either in first-order logic or, if that is not possible, in second-order logic:

- The graph G does not contain any cycles of length 2 or less. (The length of a cycle is the number of its edges.)
- The graph G does not contain any cycles.

[20] $\boxed{5}$ Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
$\neg(\phi \leftrightarrow \psi) \equiv (\phi \leftrightarrow \neg\psi)$
A propositional formula ϕ is a tautology if its negation $\neg\phi$ is not a tautology.
For every sentence ϕ in predicate logic and model $\mathcal{M}$ either $\mathcal{M} \models \phi$ or $\mathcal{M} \not\models \phi$.
For every formula in predicate logic there is an equisatisfiable formula in prenex normal form.
If a variable x occurs bound in a predicate logic formula ϕ, then x does not occur free in ϕ.
No single connective from the set $\{\bot, \neg, \vee, \wedge, \leftrightarrow, \rightarrow\}$ is adequate (on its own) for propositional logic.
If n is the length of the propositional formula ϕ, then the length of the conjunctive normal form of ϕ is at most n^2.
Can the propositional formula $p \wedge (\neg p \vee q \vee r) \wedge \neg q \wedge (q \vee \neg r)$ be shown unsatisfiable by using *only* unit propagation in DPLL?
Every prefix (i.e., initial substring) of a propositional formula does not contain more closing parentheses than opening parentheses.
If a sentence ψ in predicate logic does not have a model with infinitely many elements, then there is some natural number n such that no model of ψ has more than n elements.