Logik	WS 2016/2017	LVA 703026
EXAM 1		February 2, 2017

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 Consider the following boolean functions for $n \geqslant 1$:

$$\mathrm{HWB}_n(x_1,\dots,x_n) = \begin{cases} 0 & \text{if } \mathrm{wt}(x_1,\dots,x_n) = 0 \\ x_{\,\mathrm{wt}(x_1,\dots,x_n)} & \text{otherwise} \end{cases}$$

where $\mathrm{wt}(x_1,\dots,x_n) = \sum_{i=1}^{n} x_i$ is the number of inputs that are set to 1.

[7] (a) Give a binary decision tree for HWB_3 with the variable ordering $[x_1, x_2, x_3]$ and use the reduce algorithm to construct an equivalent reduced OBDD.

[6] (b) Compute the algebraic normal form of HWB_3.

[2] (c) Is HWB_3 monotone? Is HWB_3 self-dual?

[5] (d) Determine all minimal adequate subsets of $\{\oplus, \mathrm{HWB}_2, \rightarrow, \mathrm{HWB}_3, \neg\}$. Here $x \rightarrow y = \overline{x} + y$ and $\neg x = \overline{x}$.

[6] **2** (a) Determine whether the terms $h(x, f(x,x), y, g(y,y), z)$ and $h(a, u, u, v, v)$ are unifiable and compute a most general unifier if possible. Here a is a constant and u, v, x, y, and z are variables.

[7] (b) Transform the following formula into an equisatisfiable Skolem normal form:

$$\phi \;=\; (\forall x\, \exists y\, P(x, g(y, f(x))) \wedge \neg\forall z\, Q(z)) \vee \neg\forall x\, \forall y\, R(x,y)$$

[7] (c) Use resolution to show that the following set of clauses is unsatisfiable:

$$\begin{aligned}\{\quad &\{\neg P(x), Q(x), R(x, f(x))\},\\ &\{\neg P(x), Q(x), S(f(x))\},\\ &\{T(a)\},\\ &\{P(a)\},\\ &\{\neg R(a,z), T(z)\},\\ &\{\neg T(x), \neg Q(x)\},\\ &\{\neg T(y), \neg S(y)\}\quad\}\end{aligned}$$

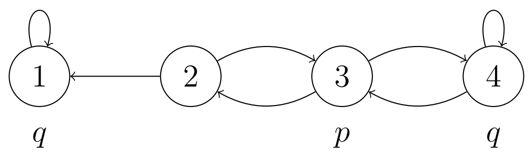
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it. (For part (c) recall that $\vee$ is right-associative.)

[7] (a) $\forall x\, P(f(x), a) \vdash \exists y\, \forall x\, P(x, y)$

[6] (b) $\exists x\, \forall y\, P(x, y) \vdash \forall y\, \exists x\, P(x, y)$

[7] (c) $\forall x\, \forall y\, \forall z\, (x < y \wedge y < z \rightarrow x < z), \forall x\, \forall y\, (x = y \vee x < y \vee y < x), a < b \vdash a < c \vee c < b$

4 Consider the following model $\mathcal{M}$:



[7] (a) Use the labelling algorithm to determine in which states of the model $\mathcal{M}$ the CTL formula $\mathsf{E}[\mathsf{EX}\, p\, \mathsf{U}\, (\mathsf{A}[p\, \mathsf{U}\, q] \vee \mathsf{AG}\, q)]$ holds.

[6] (b) Give an LTL formula ψ such that $\mathcal{M}, s \vDash \psi$ if and only if $s \in \{1, 2\}$.

[7] (c) Show that the CTL formula $\mathsf{AG}(p \rightarrow \mathsf{AF}\, \mathsf{AX}\, q)$ and the LTL formula $\mathsf{G}(p \rightarrow \mathsf{F}\, \mathsf{X}\, q)$ are not equivalent.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
The LTL formulas $\phi\, \mathsf{W}\, \psi$ and $\phi\, \mathsf{U}\, \psi \wedge \mathsf{G}\phi$ are equivalent.
The set of connectives $\{\wedge, \vee, \rightarrow\}$ is adequate for propositional logic.
In natural deduction for propositional logic $\neg\neg$e is a basic proof rule.
Tseitin's transformation transforms every formula into an equivalent CNF.
For every sentence ϕ there exists a Skolem normal form ψ such that $\phi \equiv \psi$.
If ϕ is a sentence then $\mathcal{M} \vDash_l \phi$ if and only if $\mathcal{M} \vDash_{l'} \phi$ for all environments l and l'.
The boolean function $f(x, y) = a \oplus bx \oplus cy \oplus dxy$ is affine for every $a, b, c, d \in \{0, 1\}$.
A sentence ϕ has a model with infinitely many elements if ϕ has a model with at least n elements for all $n \geqslant 1$.
Clauses C_1 and C_2 without common variables clash if there exist literals $l_1 \in C_1$ and $l_2 \in C_2$ such that l_1 and l_2 are unifiable.
To check whether a given comparator network with n wires is a sorting network, it suffices to test all 2^n sequences containing only zeroes and ones.