| Logik | WS 2016/2017 | LVA 703026 |
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| EXAM 2 | | March 2, 2017 |

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

[6] **1** (a) Use Tseitin's transformation to compute an equisatisfiable CNF of the following propositional formula:

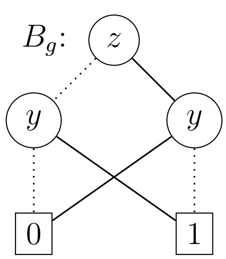
$$\phi \;=\; \neg(r \to (q \land p)) \to ((\neg p \land r) \lor q)$$

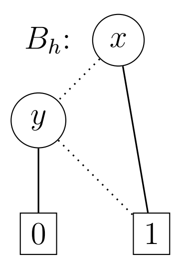
[7] (b) Give a precise definition of the backjump rule in (basic) DPLL.

[7] (c) Use DPLL to decide satisfiability of the following propositional formula:

$$\psi \;=\; (r \lor q) \land (t \lor \neg q \lor p) \land (\neg t \lor \neg q) \land (t \lor \neg p) \land (\neg r \lor q)$$

**2** Consider the boolean function $f(x, y, z) = xy \oplus z$ and the following two reduced OBDDs:




[6] (a) Construct a reduced OBDD for $f$ with variable ordering $[x, y, z]$.

[7] (b) Compute $\mathsf{apply}(\cdot, B_g, B_h)$.

[7] (c) Starting from $B_g$, compute a reduced OBDD that is equivalent to $\forall y.g$.

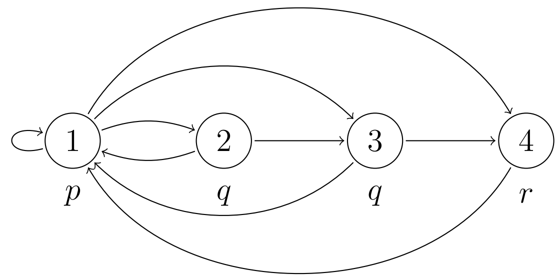
**3** For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it. (Hint: The sequent of part (b) is valid. Using LEM on $\forall y\, P(y)$ as first step does not hurt.)

[6] (a) $\vdash\; (p \to q) \lor (q \to p)$

[7] (b) $\vdash\; \exists x\, \forall y\, (P(x) \to P(y))$

[7] (c) $\vdash\; \exists x\, \forall y\, (\neg Q(y, y) \to Q(x, y))$

4 Consider the model $\mathcal{M}$




[6] (a) For each of the LTL formulas $\phi_1 = \mathsf{X}(p \,\mathsf{U}\, r)$, $\phi_2 = \mathsf{GF}p \wedge \mathsf{GF}q \wedge \mathsf{GF}r$, and $\phi_3 = \mathsf{G}(p \to \mathsf{X}q)$, find a path $\pi_i$ starting at state 1 such that $\pi_i \models \phi_i$ for each $i \in \{1, 2, 3\}$.

[7] (b) Give an LTL formula that distinguishes states 2 and 3 of $\mathcal{M}$.

[7] (c) Use the labeling algorithm to determine in which states of the model $\mathcal{M}$ the CTL formula $\psi = \mathsf{E}[q \,\mathsf{U}(\mathsf{EX}\, p \wedge \mathsf{AF}\, r)]$ holds.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

| statement |
|---|
| Peano arithmetic is decidable. |
| $\forall x\, P(x) \wedge \forall y\, Q(y) \;\vdash\; \exists x\, (P(x) \wedge \forall y\, Q(y))$ |
| Validity in propositional logic is undecidable. |
| The boolean function $\mathsf{HWB}_2(x, y)$ is self-dual. |
| Every Skolem normal form is a prenex normal form. |
| The boolean function $f(x, y, z) = \overline{(x + z)} \cdot y$ is adequate. |
| Every pair of unifiable terms has a unique most general unifier. |
| The set $\{\mathsf{AX}, \mathsf{AG}, \mathsf{AU}\}$ is an adequate set of temporal connectives for CTL. |
| A sentence in predicate logic has a finite model if it has an infinite model. |
| The set of temporal LTL connectives $\{\mathsf{U}, \mathsf{G}\}$ is adequate for the LTL fragment consisting of negation normal forms without $\mathsf{X}$. |