EXAM 3 September 28, 2017

This exam consists of <u>five</u> exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

[7] **1** (a) Use DPLL to determine whether or not the formula

$$\phi \;=\; (\neg p \vee q \vee r) \wedge (\neg r \vee \neg s \vee \neg p) \wedge p \wedge (\neg p \vee s \vee \neg r) \wedge (\neg p \vee \neg q)$$

is satisfiable.

[6] (b) Compute the algebraic normal form of the boolean function

$$f(x,y,z,u) \;=\; (\overline{x} + y + z)(\overline{z} + \overline{u} + \overline{x})x(\overline{x} + u + \overline{z})(\overline{x} + \overline{y})$$

[7] (c) Determine adequacy of the boolean function

$$g(x,y,z) \;=\; 1 \oplus xy \oplus y\overline{z}$$

[6] **2** (a) Determine whether the terms $f(x, y, g(a, a), g(z, z))$ and $f(g(w, w), g(x, x), z, y)$ are unifiable and compute a most general unifier if possible. Here a is a constant and w, x, y, and z are variables.

[7] (b) Compute a Skolem normal form of

$$\phi \;=\; \forall z\, \neg(P(a) \wedge P(z) \to \forall x\, (P(x) \wedge \exists x\, \forall y\, (P(f(x)) \to Q(x,y))))$$

[7] (c) Use resolution to determine whether the following set of clauses is satisfiable:

$$\{\{Q(a,x)\}, \{\neg P(f(f(a)))\}, \{P(a), P(x)\}, \{\neg Q(y,a), \neg P(x), P(f(x))\}\}$$

Here a is a constant and x and y are variables.

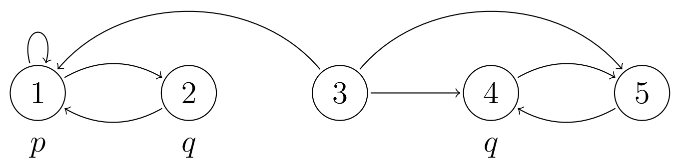
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it. Hint: In the first sequent, ϕ is a formula in which x may occur. You may use this sequent inside other proofs by instantiating ϕ and renaming variables.

[7] (a) $\neg\forall x\, \phi \vdash \exists x\, \neg\phi$

[6] (b) $\forall x\, \exists y\, (P(x,y) \to Q(y,x)) \vdash \neg\exists x\, \exists y\, (P(x,y) \wedge \neg Q(y,x))$

[7] (c) $\vdash \exists x\, (D(x) \to \forall y\, D(y))$

4 Consider the LTL formula $\phi = \mathsf{X}\, p\, \mathsf{U}\, \mathsf{G}\, \mathsf{F}\, q$, the CTL formula $\psi = \mathsf{AF}(p \vee \mathsf{AX}\, q)$, and the model $\mathcal{M}$:

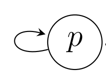


[6] (a) Determine in which states of $\mathcal{M}$ the formulas ϕ and ψ hold.

[7] (b) Transform ψ into an equivalent CTL formula that only uses the temporal connectives EX, EG, and EU.

[7] (c) For each $i \in \{1, 2, 3, 4, 5\}$ find a CTL formula χ_i that holds only in state i of $\mathcal{M}$.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
Every boolean function has a unique ANF.
Every propositional formula has a unique equivalent CNF.
The formula $\forall x\, Q(x, a)$ is a Skolem normal form of $\forall x\, \exists y\, Q(x, y)$.
The CTL formula $p \vee \mathsf{AX}\, \mathsf{AF}\, p$ is equivalent to the LTL formula $\mathsf{F}p$.
The LTL formula $p \wedge \mathsf{X}\, p$ is satisfied by all paths in the model .
The set $\{\mathsf{EX}, \mathsf{EG}, \mathsf{EU}\}$ is an adequate set of temporal connectives for CTL.
Not every valid propositional sequent has a finite natural deduction proof.
We have $\phi \to \psi \to \chi \equiv \phi \wedge \psi \to \chi$ for arbitrary propositional formulas ϕ, ψ, and χ.
Applying the unification algorithm from the lecture to $x \approx f(x)$ yields the result $\{x \mapsto f(x)\}$.
Every sentence that holds in the standard model of arithmetic is provable in Peano Arithmetic.