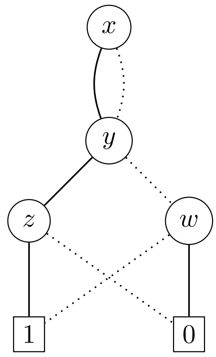
universität innsbruck

Logik	WS 2017/2018	LVA 703027

EXAM 1 — February 1, 2018

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 This question concerns the boolean function f defined by $f(x,y,z,w) = yz \oplus \overline{y}\,\overline{w}$ with BDD B_f:



[7] (a) Explain why B_f is not reduced, and give reduced OBDDs for f for the following variable orderings: $[x,y,z,w]$, $[w,z,y,x]$, and $[x,y,w,z]$.

[6] (b) Give an expression only using f and the variables x and y that represents the function xy, and verify that your representation is correct by computing the algebraic normal form. (Hint: first evaluate $f(x,x,x,x)$.)

[7] (c) Show by means of Post's adequacy theorem that $\{f\}$ is not adequate, but that $\{f,\neg\}$ is adequate.

[6] **2** (a) Use the unification algorithm from the lecture to determine whether the two terms $f(x, g(h(y,c)))$ and $f(h(c,y), g(x))$ are unifiable and compute a most general unifier if possible. Note that c is a constant while x and y are variables.

[7] (b) Transform the following formula into an equisatisfiable Skolem normal form:

$$\phi = (\exists x\,\forall y\,\neg P(x,y)) \to (\forall y\,\exists x\,\neg P(x,y))$$

[7] (c) Use resolution to determine whether the following set of clauses is satisfiable (where a is a constant):

$$\{\{\neg P(a,y), \neg P(a,z)\}, \{P(u, f(y,u))\}\}$$

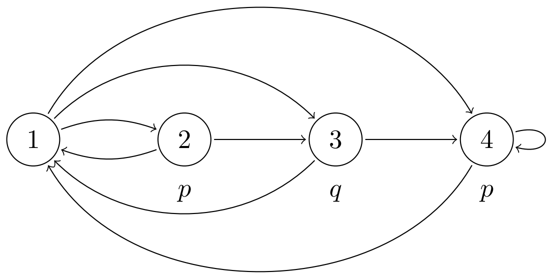
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[7] (a) $\forall x\,(\neg P(x) \to \neg\exists y\,(x = 2\cdot y)), a = 2\cdot b, \exists y\,\forall x\,(P(x) \to Q(x,y)) \;\vdash\; \exists y\,Q(a,y)$

[7] (b) $\forall x\,\forall y\,(P(x,y) \to Q(y)), \forall x\,\forall y\,(P(x,y) \to P(y,x)) \;\vdash\; \forall x\,\forall y\,(P(x,y) \to Q(x))$

[6] (c) $\forall x\,\exists y\,P(x,y), \forall x\,\forall y\,(P(x,y) \to P(y,x)) \;\vdash\; \exists x\,\forall y\,P(x,y)$

4 Consider the model $\mathcal{M}$



and the CTL formula $\phi = \mathsf{E}[p\,\mathsf{U}\,(\mathsf{EX}\,q \vee \mathsf{AF}\,\mathsf{A}[p\,\mathsf{U}\,q])]$.

[7] (a) Use the labeling algorithm to determine in which states ϕ holds.

[7] (b) For each state $i \in \{1, 2, 3, 4\}$ give an LTL formula ψ_i that holds only in state i.

[6] (c) Transform ϕ into an equivalent CTL formula that uses only temporal connectives from $\{\mathsf{EU}, \mathsf{AX}, \mathsf{AF}, \mathsf{EG}\}$.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
Presburger arithmetic is consistent.
There are no infinite derivations in DPLL with learning.
The terms $f(x, x, y, z)$ and $f(y, g(z), g(z), z)$ are unifiable.
The CTL* formulas $\mathsf{E}[\mathsf{G}\,\mathsf{F}\,p]$ and $\mathsf{E}[\mathsf{G}\,\mathsf{E}[\mathsf{F}\,p]]$ are equivalent.
Sorting n inputs with bitonic sort requires $\mathcal{O}(n \log n)$ comparators.
The clause $\{q\}$ is a resolvent of the clauses $\{\neg p, q, r\}$ and $\{p, q, \neg r\}$.
The sequent $\vdash\ \forall x\, \exists y\, (P(x) \to Q(y)) \to \forall x\, (P(x) \to \exists y\, Q(y))$ is valid.
Intuitionistic logicians do not use the proof rule $\neg\neg$i of natural deduction.
Every n-ary boolean function that admits a reduced OBDD with $n - 1$ internal nodes is affine.
If $\phi_1, \phi_2 \vDash \psi$ then the sequent $\vdash \neg\psi \to \neg\phi_1 \vee \neg\phi_2$ is valid, for all propositional formulas ϕ_1, ϕ_2, and ψ.