Logik | WS 2017/2018 | LVA 703027

EXAM 2 — February 28, 2018

This exam consists of <u>five</u> exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 Consider the boolean function $f(x, y, z) = x \oplus (y + z)$.

[7] (a) Is f monotone? Is f self-dual? Is f affine?

[6] (b) Can $\overline{x}$ be expressed (only) using f?

[7] (c) Let g be a boolean function such that $\{g\}$ is adequate. Is $\{\hat{g}\}$ adequate? Here $\hat{g}$ is the dual of g.

[7] **2** (a) Use the unification algorithm from the lecture to determine whether the two terms $h(f(x, z), b, f(z, y))$ and $h(y, z, f(b, f(a, w)))$ are unifiable and compute a most general unifier if possible. Note that a and b are constants while w, x, y and z are variables.

[5] (b) Explain how resolution can be used to determine the validity of a formula ϕ in predicate logic. You do not have to explain resolution.

[8] (c) Use resolution to determine whether the formula

$$\phi = \neg\forall x\, \exists y\, \neg P(x, y) \wedge (\forall z\, \exists w\, P(w, z) \to \forall z'\, Q(z'))$$

is satisfiable.

3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

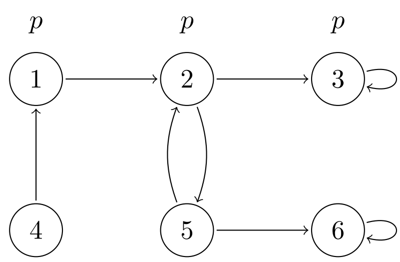
[6] (a) $p \vee q, \neg r \to \neg p \;\vdash\; q \vee r$

[7] (b) $\exists x\, P(x), x = y \;\vdash\; P(y)$

[7] (c) $\vdash\; \forall x\, ((\neg P(x) \to \bot) \to P(x))$

4 Consider the CTL formula $\phi = \mathsf{AX}(\mathsf{AF}\, p \vee \mathsf{AG}\, \neg p)$

[7] (a) Use the labeling algorithm to determine in which states ϕ holds, for the following model:



[7] (b) Give a CTL formula ψ that is equivalent to ϕ but does not use the temporal connectives in ϕ (so ψ may not contain AX, AF, and AG).

[6] (c) Given a path π in a model $\mathcal{M}$, define $\pi \models \mathsf{G}\varphi$, i.e., satisfaction of the LTL formula $\mathsf{G}\varphi$ with respect to π. Using the definition, show that $\mathsf{G}\varphi \equiv \mathsf{GG}\varphi$ for every LTL formula φ.

[20] $\boxed{5}$ Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
The dual of any affine boolean function is monotone.
The symbols ζ and Γ are letters from the Greek alphabet.
If $F = (\neg p \vee q \vee r) \wedge \neg q$ then $\overset{d}{p} \parallel F \implies \overset{d}{p}\, r \parallel F$ in DPLL.
The LTL formulas $\phi \mathsf{R} \psi$ and $\psi \mathsf{U} (\phi \wedge \psi) \vee \mathsf{G} \psi$ are equivalent.
The sequent $\neg\forall x\, (P(x) \vee Q(y)) \vdash \neg Q(y) \wedge \exists x\, \neg P(x)$ is valid.
If f is a binary boolean function then $f(x,y) = \overline{y} \cdot f(x,0) \oplus y \cdot f(x,1)$.
Sorting n inputs with insertion sort requires at most $2n+3$ comparators.
If $\phi \vdash \psi$ is a valid sequent in propositional logic then $\neg\phi \vee \psi$ is a tautology.
All occurrences of the variable x in the formula $P(y) \wedge \forall y\, (\exists x\, Q(x,y) \vee P(x))$ are bound.
If X is the set of all self-dual boolean functions, then any function constructed from functions in X is also self-dual.