universität innsbruck

Logik	WS 2017/2018	LVA 703027

EXAM 3 — September 27, 2018

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

[7] **1** (a) Consider the propositional formula $\varphi = \neg(p \vee q) \rightarrow ((r \wedge q) \rightarrow \neg p)$. Use Tseitin's transformation to transform φ into an equisatisfiable CNF.

[5] (b) Use DPLL to test the satisfiability of the propositional formula

$$\phi \;=\; (s \vee \neg t) \wedge p \wedge (t \vee \neg p \vee r) \wedge (\neg r \vee q) \wedge (\neg q \vee t) \wedge (\neg s \vee \neg t)$$

[8] (c) Consider the propositional formula

$$\psi \;=\; \underbrace{(\neg 1 \vee 2)}_{\alpha} \wedge \underbrace{(\neg 1 \vee 3)}_{\beta} \wedge \underbrace{(\neg 4 \vee 5)}_{\gamma} \wedge \underbrace{(\neg 5 \vee \neg 6)}_{\delta} \wedge \underbrace{(\neg 2 \vee \neg 3 \vee 6)}_{\epsilon}$$

and the following (partial) DPLL derivation ending in a conflict:

$$\begin{array}{lrcll}
 & & \| & \psi & \\
\Longrightarrow & \overset{d}{1} & \| & \psi & \text{(decide)} \\
\Longrightarrow & \overset{d}{1}\ 2 & \| & \psi & \text{(unit propagation)} \\
\Longrightarrow & \overset{d}{1}\ 2\ 3 & \| & \psi & \text{(unit propagation)} \\
\Longrightarrow & \overset{d}{1}\ 2\ 3\ \overset{d}{4} & \| & \psi & \text{(decide)} \\
\Longrightarrow & \overset{d}{1}\ 2\ 3\ \overset{d}{4}\ 5 & \| & \psi & \text{(unit propagation)} \\
\Longrightarrow & \overset{d}{1}\ 2\ 3\ \overset{d}{4}\ 5\ \neg 6 & \| & \psi & \text{(unit propagation)}
\end{array}$$

Draw the conflict graph, determine all unique implication points with corresponding backjump clauses, and give the result obtained by applying the backjump rule using <u>one</u> of those clauses.

[6] **2** (a) For a signature only containing a binary symbol f, give two non-unifiable terms s and t over the signature (so only containing f and variables), such that s and t do not have variables in common. Prove non-unifiability of s and t by means of the unification algorithm.

[7] (b) Use resolution to show that the formula

$$\phi \;=\; \exists x\,(P(x) \wedge P(a)) \vee \exists x\,(\neg P(x) \wedge \neg P(a))$$

is valid. Here a is a constant.

[7] (c) For the formula

$$\psi \;=\; \forall x\,\forall y\,\forall z\,(R(x,y) \wedge R(x,z) \rightarrow R(y,z))$$

give models $\mathcal{M}$ and $\mathcal{M}'$ such that $\mathcal{M} \models \psi$ but $\mathcal{M}' \not\models \psi$, and the interpretation of the binary predicate symbol is not the empty set.

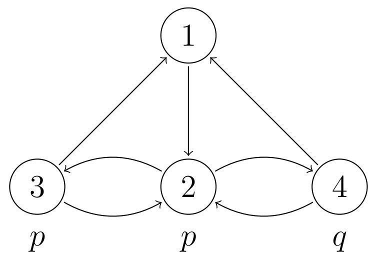
[3] For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[7] (a) $\exists x\, \neg Q(x) \ \vdash\ \neg\forall y\, Q(y)$

[7] (b) $\forall x\, (P(x) \rightarrow Q(f(x))), \forall x\, (Q(x) \rightarrow \neg P(x)), P(f(c)), \neg Q(c) \ \vdash\ \forall x\, (P(x) \vee Q(x))$

[6] (c) $\forall x\, (P(x) \vee Q(x)), \forall x\, (R(x) \rightarrow \neg P(x)) \ \vdash\ \exists x\, \neg Q(x) \rightarrow \exists x\, \neg R(x)$

[4] Consider the model $\mathcal{M}$



[7] (a) Give an LTL formula ϕ such that $\pi \models \phi$ if and only if $\pi = 1 \rightarrow 2 \rightarrow 3 \rightarrow 2 \rightarrow 3 \rightarrow \cdots$.

[6] (b) For each of the LTL formulas $\psi_1 = \mathsf{G}(\mathsf{F}\, p \wedge \mathsf{F}\, q)$, $\psi_2 = \mathsf{F}(p \rightarrow \mathsf{X}\, q)$, and $\psi_3 = \mathsf{G}(p \vee \mathsf{X}\,\mathsf{X}\, q)$, find a path π_i starting at state 1 such that $\pi_i \models \psi_i$ for each $i \in \{1, 2, 3\}$.

[7] (c) Use the labeling algorithm to determine in which states of the model $\mathcal{M}$ the CTL formula $\chi = \mathsf{E}[p\, \mathsf{U}\, (\mathsf{EX}\, p \wedge \neg\, \mathsf{AF}\, q)]$ holds.

[20] [5] Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
There exists an OBDD with exactly two nodes.
The boolean function $f(x, y) = (x \oplus y) + \overline{x}$ is affine.
Every adequate set of LTL connectives contains F or U.
Resolution for predicate logic without factoring is sound.
The propositional formula $(p \wedge \neg q) \vee \neg(q \leftrightarrow r) \vee \neg p \vee r$ is valid.
Tseitin's transformation transforms every CNF into a satifiable CNF.
The sequent $\neg P \rightarrow \neg(S \vee R), \exists x\, Q(x) \rightarrow T \vdash S \vee \neg T \rightarrow P \vee \forall x\, \neg Q(x)$ is valid.
The set $\{\forall x\, P(x), \exists x\, (P(x) \rightarrow \neg P(x))\}$ of predicate logic sentences is consistent.
Every n-ary affine boolean function admits an OBDD with at most $n + 2$ nodes.
If basic DPLL without decide concludes satisfiability of a formula F then F is a tautology.