Logik	WS 2018/2019	LVA 703027

EXAM 1	January 28, 2019

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 Consider the boolean function c given by

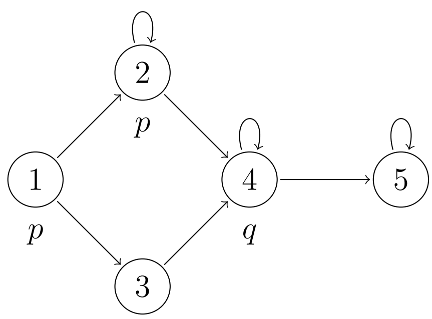
$$\mathsf{c}(x, y, z) = \begin{cases} 1 & \text{if } x + y + z > 1 \\ 0 & \text{otherwise} \end{cases}$$

[7] (a) Use the reduce algorithm from the lecture to compute a reduced OBDD for c with variable ordering $[x, y, z]$.

[6] (b) Compute the algebraic normal form of c.

[7] (c) Determine which of the five properties from Post's adequacy theorem hold for c. Conclude whether or not the set $\{\mathsf{c}\}$ is adequate.

2 Consider the model $\mathcal{M}$:



[7] (a) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}$ the CTL formula $\phi = \mathsf{AG}(\mathsf{E}[p \, \mathsf{U} \, \mathsf{AF}\, q])$ holds.

[7] (b) Consider the LTL formulas $\phi_1 = p \, \mathsf{U} \, q$, $\phi_2 = \mathsf{G}\, p \vee \mathsf{X}\, \mathsf{G}\, \neg p$, and $\phi_3 = \mathsf{G}(p \vee \mathsf{X}\, \neg p)$. Determine for $1 \leqslant i \leqslant 3$ in which states of $\mathcal{M}$ the formula ϕ_i holds.

[6] (c) Give a CTL formula without the connectives EF and EG that is equivalent to $\mathsf{EF}\,\mathsf{EG}\, p$.

3 Consider the following CNF φ:

$$\overset{\alpha}{(4 \vee \neg 3 \vee \neg 1)} \wedge \overset{\beta}{(\neg 5 \vee \neg 2 \vee \neg 3)} \wedge \overset{\gamma}{(\neg 3 \vee 6)} \wedge \overset{\delta}{(\neg 4 \vee \neg 6 \vee \neg 7)} \wedge \overset{\epsilon}{(8 \vee \neg 6)} \wedge \overset{\zeta}{(\neg 4 \vee \neg 8 \vee 7)}$$

[7] (a) Show that the sequence of decisions 1, 2, 3 followed by applications of unit propagate leads to a conflict, and construct the conflict graph.

[6] (b) Starting from the conflict clause in part (a), use resolution to compute the backjump clause corresponding to the first UIP. What is the result of backjump using that clause?

[7] (c) Use the unification algorithm from the lecture to determine whether the two terms $f(x, f(g(x), y))$ and $f(g(y), f(g(z), z))$ are unifiable and compute a most general unifier if possible. Here x, y and z are variables.

4 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[7] (a) $\exists x\, \exists y\, P(x, y),\ \neg\exists x\, P(x, x)\ \vdash\ \exists x\, \exists y\, \neg(x = y)$

[7] (b) $\forall x\, \forall y\, (P(x, y) \wedge P(y, x) \rightarrow (x = y))\ \vdash\ \forall x\, P(x, x)$

[6] (c) $p \rightarrow q,\ q \rightarrow p\ \vdash\ p \vee q \rightarrow p \wedge q$

[20] **5** Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
Satisfiability in predicate logic is undecidable.
The dual of an affine boolean function is affine.
The clause $\{P(b, a)\}$ is a factor of $\{P(x, a), \neg P(b, y)\}$.
The empty clause is a resolvent of $\{p, \neg q\}$ and $\{\neg p, q\}$.
The boolean function $f(x, y, z) = \overline{(\overline{x} + y) \cdot (x + z)}$ is adequate.
The CTL* formulas $\mathsf{A}[\mathsf{X}\, p \vee \mathsf{X}\,\mathsf{X}\, p]$ and $\mathsf{AX}\, p \vee \mathsf{AX}\,\mathsf{AX}\, p$ are equivalent.
The term $f(x, y)$ is free for y in $\forall x\, \exists y\, (P(x) \rightarrow Q(y)) \rightarrow \forall x\, (P(y) \rightarrow \exists y\, Q(x))$.
The set $[\![\, \mathsf{EG}\, \phi \,]\!]$ is the least fixed point of the function $F_{\mathsf{EG}}(X) = [\![\, \phi \,]\!] \cap \mathsf{pre}_{\exists}(X)$.
If $\mathcal{M}, s \not\models \neg\phi$ then $\mathcal{M}, s \models \phi$, for all LTL formulas ϕ, models $\mathcal{M} = (S, \rightarrow, L)$, and states $s \in S$.
To determine the satisfiability of a CNF consisting of clauses having exactly two literals with DPLL, the decide rule needs to be used at most once.