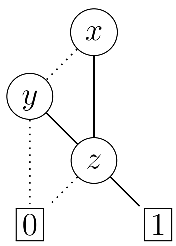
Logik | WS 2018/2019 | LVA 703027

EXAM 2 | February 28, 2019

This exam consists of <u>five</u> exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 Consider the boolean function f defined by $f(x, y, z) = xz + yz$ with OBDD B_f:

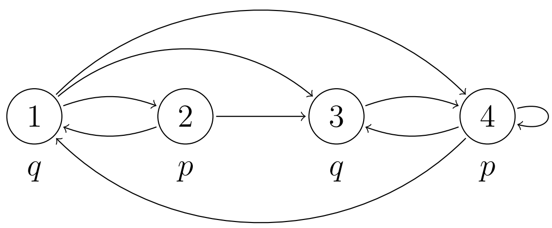


[7] (a) Give reduced OBDDs for f with variable orderings $[y, x, z]$ and $[z, x, y]$.

[7] (b) Compute a reduced OBDD for $\exists y.f$.

[6] (c) Give an expression only using f and the variables x and y that is equivalent to the boolean function xy.

2 Consider the model $\mathcal{M}$



[7] (a) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}$ the CTL formula $\phi = \mathsf{E}[(\mathsf{AX}\,\neg p)\,\mathsf{U}\,\mathsf{A}[q\,\mathsf{U}\,(\mathsf{EG}\,\neg q)]]$ holds.

[7] (b) Give an LTL formula ψ such that $\mathcal{M}, s \models \psi$ if and only if $s \in \{2, 3\}$.

[6] (c) Are the CTL* formulas $\mathsf{E}[\mathsf{X}\,\mathsf{F}\,\mathsf{G}\,\neg p]$ and $\neg\mathsf{A}[\mathsf{X}\,\mathsf{G}\,\mathsf{E}[\mathsf{F}\,p]]$ equivalent?

[7] 3 (a) Consider the following CNF φ:

$$(\neg 1 \vee \overset{\alpha}{\neg 3} \vee 4) \wedge (\neg 2 \vee \overset{\beta}{\neg 3} \vee \neg 5) \wedge (\neg 3 \overset{\gamma}{\vee} 6) \wedge (\neg 4 \vee \overset{\delta}{\neg 6} \vee \neg 7) \wedge (\neg 6 \overset{\epsilon}{\vee} 8) \wedge (\neg 4 \vee \overset{\zeta}{7} \vee \neg 8)$$

Starting from the state $\overset{d}{1}\,\overset{d}{2}\,\overset{d}{3} \parallel \varphi$, use DPLL to determine satisfiability of φ.

[7] (b) Transform the formula $\forall x\, \exists y\, \forall z\, \exists w\, (P(x,w) \rightarrow H(y) \wedge \neg Q(z,w))$ into clausal form.

[6] (c) Use resolution to show unsatisfiability of the following clausal form.

$$\{\ \{\neg P(x, f(x)), Q(f(x))\},\ \{\neg Q(x), R(f(x))\},\ \{\neg R(f(f(f(x))))\},$$
$$\{P(a, f(a))\},\ \{P(f(f(f(a))), y)\}\ \}$$

For each resolvent give the mgu for the clashing literals. Here a is a constant.

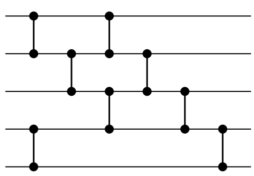
4 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it. In parts (b) and (c), k and m are constants.

[7] (a) $(p \rightarrow q) \rightarrow r,\ q\ \vdash\ p \vee r$

[7] (b) $\forall x\, (P(x,m) \rightarrow \neg Q(x)),\ Q(m)\ \vdash\ Q(k) \rightarrow \neg P(k,m)$

[6] (c) $\forall x\, (P(x,m) \rightarrow \neg Q(x)),\ Q(m)\ \vdash\ Q(k) \rightarrow \neg(k = m)$

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
If f is a boolean function then $f[0/x] \cdot f[1/x] = 0$.
The PCP instance $(1, 11)$, $(01, 10)$, $(00, 0)$ has a solution.
The boolean function $f(x,y,z) = \overline{(\overline{x} \oplus y) \cdot (x + z)}$ is monotone.
The propositional formula $p \vee q$ has an equisatisfiable Horn formula.
Every LTL formula is semantically equivalent to a CTL* path formula.
$P(f(y)) \rightarrow \forall x\, \exists y\, (\neg Q(x,y) \vee P(y)) \ \dashv\vdash\ \forall x\, (P(f(y)) \rightarrow \exists y\, (\neg Q(x,y) \vee P(y)))$
The CTL formulas $\mathsf{A}[\phi\, \mathsf{U}\, \psi]$ and $\neg(\mathsf{E}[\neg\psi\, \mathsf{U}\, (\neg\psi \wedge \neg\phi)] \vee \neg\, \mathsf{AF}\, \psi)$ are equivalent.
Any valid sequent has a natural deduction proof in which the proof rule $\neg\neg$e is not used.
The natural numbers can be colored with two colors such that not all of x, y, z have the same color whenever $x^2 + y^2 = z^2$ with $1 \leqslant x, y, z \leqslant 100$.
The comparator network  is a sorting network.