universität innsbruck

Logik	WS 2018/2019	LVA 703027

EXAM 3	September 26, 2019

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 In this exercise we consider the boolean function ite defined by

$$\mathsf{ite}(x,y,z) = \begin{cases} z & \text{if } x = 0 \\ y & \text{if } x = 1 \end{cases}$$

[7] (a) Compute a reduced OBDD for ite with variable ordering $[y, x, z]$.

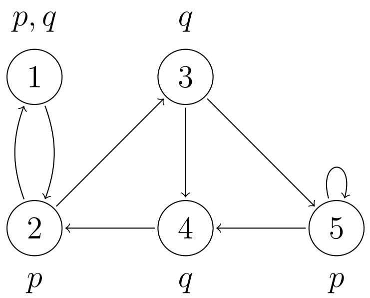
[7] (b) Find the missing clauses such that $a \leftrightarrow \mathsf{ite}(x,y,z)$ and

$$(a \vee \neg x \vee \neg y) \;\wedge\; \boxed{} \;\wedge\; \boxed{} \;\wedge\; (\neg a \vee x \vee z)$$

are equivalent.

[6] (c) Is ite monotone? Is ite self-dual? Is ite affine?

2 Consider the model $\mathcal{M}$:



[7] (a) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}$ the CTL formula $\phi = \mathsf{A}[p \,\mathsf{U}\, \mathsf{AF}\, q] \wedge \mathsf{AX}\,\mathsf{EF}\,\neg p$ holds.

[6] (b) Consider the LTL formulas $\phi_1 = \mathsf{G}(\mathsf{F}\,p \wedge \mathsf{F}\,q)$, $\phi_2 = p \,\mathsf{U}\, q \to \neg p$, and $\phi_3 = \mathsf{G}(p \vee \mathsf{X}\,\mathsf{X}\,q)$. Find a model $\mathcal{M}$ and a state s such that $\mathcal{M}, s \models \phi_i$ for all $1 \leqslant i \leqslant 3$.

[7] (c) Show that the LTL formula $\mathsf{F}\,p \to \mathsf{F}\,q$ and the CTL formula $\mathsf{AF}\,p \to \mathsf{AF}\,q$ are not equivalent.

[7] 3 (a) Use resolution to determine whether the formula $(p \to q \to p) \to p$ is valid or not.

[7] (b) Give a Skolem normal form of the formula $\exists x\, ((\exists y\, P(x,y)) \to (\forall y\, \exists z\, Q(y,z)))$.

[6] (c) Determine whether the terms $h(f(x,x), y, f(a,x))$ and $h(y, f(x,a), y)$ are unifiable and compute a most general unifier if possible. Here a is a constant and x and y are variables.

4 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $p \to t,\ \neg t \vee \neg s\ \vdash\ p \to \neg s$

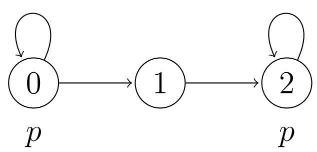
[7] (b) $Q(c),\ \exists x\, (Q(x) \to \forall x\, P(x))\ \vdash\ P(c)$

[7] (c) $\forall x\, Q(x),\ \exists x\, (Q(x) \to \forall x\, P(x))\ \vdash\ P(c)$

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
The sequent $\vdash \forall x\, (D(x) \to \exists y\, D(y))$ is valid.
Every binary monotone boolean function is affine.
The propositional formulas $\bot$ and $\top$ are equisatisfiable.
The LTL formulas $\phi\, \mathsf{R}\, \psi$ and $\mathsf{G}\, \psi \vee \psi\, \mathsf{U}\, (\phi \wedge \psi)$ are equivalent.
The hereditary base 3 representation of 333 is $3^{3+2} + 3^{3+1} + 3^2$.
The formula $\forall x\, \forall y\, P(x,y) \vee \neg Q(f(x))$ is a Skolem normal form.
The set $\{\, \forall x\, \exists y\, P(x,y),\ \neg\exists x\, \forall y\, \neg P(y,x),\ \forall x\, \neg P(x,x) \,\}$ is consistent.
Every sorting network for $n > 1$ wires has at least $(n-1)^2$ comparators.
The substitution $\{x \mapsto y, y \mapsto x\}$ is a most general unifier of $f(x,y)$ and $f(y,x)$.

In the model $\mathcal{M}$



$\mathcal{M}, 0 \vDash \mathsf{AF}\, \mathsf{AG}\, p$ holds.