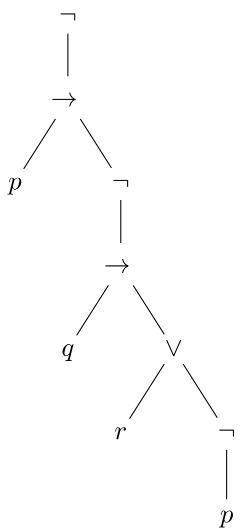
universität innsbruck

Logik	SS 2024	LVA 703026 + 703027
Week 1		March 7, 2024

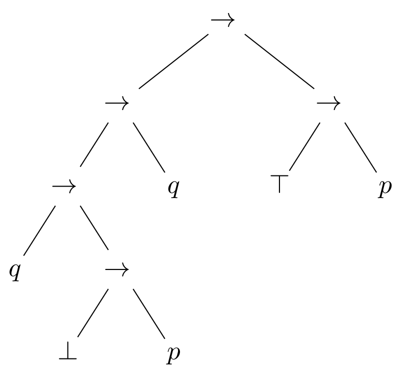
Selected Solutions

2 (a) For $\varphi = \neg(p \to \neg(q \to r \vee \neg p))$ we have



with 9 subformulas: φ, $p \to \neg(q \to (r \vee \neg p))$, p, $\neg(q \to (r \vee \neg p))$, $q \to (r \vee \neg p)$, q, $r \vee \neg p$, r and $\neg p$.

For $\psi = ((q \to (\bot \to p)) \to q) \to \top \to p$ we have



with 9 subformulas: ψ, $(q \to (\bot \to p)) \to q$, $q \to (\bot \to p)$, q, $\bot \to p$, $\bot$, p, $\top \to p$ and $\top$.

(b) For $\varphi = \neg(p \to \neg(q \to r \vee \neg p))$ we have

p	q	r	$\neg p$	$r \vee \neg p$	$q \to r \vee \neg p$	$\neg(q \to r \vee \neg p)$	$p \to \neg(q \to r \vee \neg p)$	φ
T	T	T	F	T	T	F	F	T
T	T	F	F	F	F	T	T	F
T	F	T	F	T	T	F	F	T
T	F	F	F	F	T	F	F	T
F	T	T	T	T	T	F	T	F
F	T	F	T	T	T	F	T	F
F	F	T	T	T	T	F	T	F
F	F	F	T	T	T	F	T	F

For ψ we have

p	q	$\bot$	$\bot \to p$	$q \to (\bot \to p)$	$(q \to (\bot \to p)) \to q$	$\top$	$\top \to p$	ψ
T	T	F	T	T	T	T	T	T
T	F	F	T	T	F	T	T	T
F	T	F	T	T	T	T	F	F
F	F	F	T	T	F	T	F	T

(c) Since the last column in the truth table for φ contains T, we infer that φ is satisfiable. For instance, for the valuation $v(p) = v(q) = v(r) =$ T we have $\overline{v}(\varphi) =$ T. The formula φ is not valid since the last column in the truth table contains F. For the valuation $v(p) = v(q) =$ T and $v(r) =$ F we have $\overline{v}(\varphi) =$ F.

The same holds for ψ. For instance, for the valuation $v(p) = v(q) =$ T we have $\overline{v}(\psi) =$ T and for the valuation $v(p) =$ F and $v(q) =$ T we have $\overline{v}(\psi) =$ F.

3 (a) The semantic entailment $(p \to q) \to p, \neg(q \wedge p) \vDash \neg(\neg p \to q)$ is false:

p	q	$p \to q$	$(p \to q) \to p$	$q \wedge p$	$\neg(q \wedge p)$	$\neg p$	$\neg p \to q$	$\neg(\neg p \to q)$
T	T	T	T	T	F			
T	F	F	T	F	T	F	T	F
F	T	T	F					
F	F	T	F					

(b) The semantic entailment $\neg p \wedge \neg\neg(\neg p \to \top) \vDash \bot$ is false:

p	$\neg p$	$\top$	$\neg p \to \top$	$\neg(\neg p \to \top)$	$\neg\neg(\neg p \to \top)$	$\neg p \wedge \neg\neg(\neg p \to \top)$	$\bot$
T	F	T	T	F	T	F	
F	T	T	T	F	T	T	F

4 (a) Using the procedure from slide 41 we obtain

$$\begin{aligned} p \vee ((q \vee \neg r) \wedge (p \vee (q \wedge r))) &\overset{③}{\longrightarrow} (p \vee (q \vee \neg r)) \wedge (p \vee (p \vee (q \wedge r))) \\ &\overset{③}{\longrightarrow} (p \vee (q \vee \neg r)) \wedge (p \vee ((p \vee q) \wedge (p \vee r)) \\ &\overset{③}{\longrightarrow} (p \vee (q \vee \neg r)) \wedge ((p \vee (p \vee q)) \wedge (p \vee (p \vee r))) \end{aligned}$$

(b) For instance

$$\begin{aligned}
\neg(p \to (q \land (\neg p \to q))) &\overset{①}{\longrightarrow} \neg(\neg p \lor (q \land (\neg p \to q))) \\
&\overset{①}{\longrightarrow} \neg(\neg p \lor (q \land (\neg\neg p \lor q))) \\
&\overset{②}{\longrightarrow} \neg(\neg p \lor (q \land (p \lor q))) \\
&\overset{②}{\longrightarrow} \neg\neg p \land \neg(q \land (p \lor q)) \\
&\overset{②}{\longrightarrow} p \land \neg(q \land (p \lor q)) \\
&\overset{②}{\longrightarrow} p \land (\neg q \lor \neg(p \lor q)) \\
&\overset{②}{\longrightarrow} p \land (\neg q \lor (\neg p \land \neg q)) \\
&\overset{③}{\longrightarrow} p \land ((\neg q \lor \neg p) \land (\neg q \lor \neg q))
\end{aligned}$$

5 (a) For each row whose last entry is T, we form the conjunction of the literals corresponding to the atoms. Here "corresponding" means that atom p is turned into $\neg p$ when p is assigned F, and stays p if p is assigned T. Then we form the disjunction of all conjunctions. If there are no rows ending in T we return the DNF $\bot$. If there are no atoms, we return $\top$ if the formula evaluates to T and $\bot$ if it evaluates to F.

Applying the above procedure to the truth tables computed in the solution of Exercise 2(b) yields

$$(p \land q \land r) \lor (p \land \neg q \land r) \lor (p \land \neg q \land \neg r)$$

for φ and

$$(p \land q) \lor (p \land \neg q) \lor (\neg p \land \neg q)$$

for ψ.