universität innsbruck

| Logik | SS 2024 | LVA 703026 + 703027 |
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| Week 2 | | March 14, 2024 |

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. Solutions for bonus exercises must be submitted separately. The (strict) deadline is 7 am on March 14.

## Exercises

⟨2⟩ 1. Translate the following formulas into equivalent CNFs and DNFs. Which of these formulas are satisfiable? Which are valid?

(a) $p \to (\neg q \to p)$

(b) $\neg(p \wedge q) \wedge r \to (\neg p \to \neg r)$

⟨2⟩ 2. Find the missing clauses such that the formula $a \leftrightarrow ((p \to q) \wedge (\neg p \to r))$ and the CNF

$$(\neg a \vee \neg p \vee q) \;\wedge\; \square \;\wedge\; \square \;\wedge\; (\neg a \vee p \vee r)$$

are equivalent.

⟨2⟩ 3. Determine the satisfiability of the following Horn formulas.

(a) $(p \to q) \wedge (q \wedge r \wedge p \to t) \wedge (\top \to p) \wedge (p \wedge q \to \bot)$

(b) $(\top \to p) \wedge (\top \to q) \wedge (r \to \bot) \wedge (s \wedge p \wedge q \to t) \wedge (t \to q) \wedge (\top \to t) \wedge (t \to s)$

⟨2⟩ 4. (a) Extend the lemma on slide 28 with a fourth item for implication by transforming the formula

$$\varphi \leftrightarrow (\psi \to \chi)$$

into an equivalent CNF consisting of at most three clauses.

(b) Use Tseitin's transformation to compute an equisatisfiable CNF of the formula

$$((p \wedge r) \to q) \vee \neg(q \vee \neg p)$$

⟨2⟩ 5. Consider the map of Austria

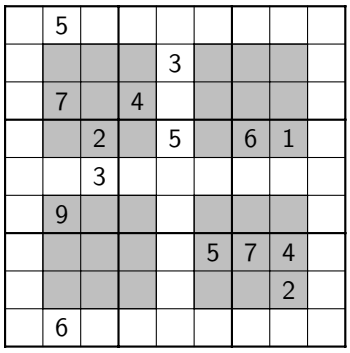


(a) Construct a CNF formula that is satisfiable if and only if the map of Austria is 3-colorable.

(b) Encode the formula of part (a) into DIMACS format and use a SAT solver to determine whether Austria is 3-colorable.

## Bonus Exercise

6. Consider Hyper Sudoku, where in addition to the usual rules each gray area must contain the digits 1 to 9:




⟨2⟩ (a) Construct a CNF formula that is satisfiable if and only if the above puzzle has a solution.

⟨3⟩ (b) Write a program that solves Hyper Sudoku puzzles. The input consists of a text file with 9 lines, each consisting of 9 digits in the range 0 – 9, where a 0 indicates an empty cell. The output should consist of 9 lines consting of 9 digits in the range 1 – 9.