universität innsbruck

Logik	SS 2024	LVA 703026 + 703027
Week 3		March 21, 2024

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. The (strict) deadline is 7 am on March 21.

Exercises

⟨3⟩ 1. In this exercise we consider the question whether there exists a blue/red coloring of the set of numbers $\{1, \ldots, n\}$ such that there is no monochromatic solution of the equation $a + b = c$ for $1 \leqslant a < b < c \leqslant n$. For instance, for $n = 5$ the answer is yes because the only solutions of the equation are

$$1 + 2 = 3 \qquad 1 + 3 = 4 \qquad 1 + 4 = 5 \qquad 2 + 3 = 5$$

and so we can (e.g.) use the coloring 1 2 3 4 5:

$$1 + 2 = 3 \qquad 1 + 3 = 4 \qquad 1 + 4 = 5 \qquad 2 + 3 = 5$$

(a) Show that the answer is yes when $n \leqslant 8$.

(b) Construct a CNF formula φ such that satisfiability of φ answers the question for $n = 9$, and encode φ into DIMACS format. Use a SAT solver to obtain the answer.

⟨2⟩ 2. Prove the validity of the following sequents using natural deduction:

(a) $p \to (q \to r), q \vdash p \to (q \wedge r)$

(b) $p \to q \vdash (p \wedge q \to p) \wedge (p \to p \wedge q)$

⟨3⟩ 3. Prove that the following propositional formulas are theorems, using natural deduction:

(a) $q \to (p \to (p \to (q \to p)))$

(b) $((p \to q) \to q) \to ((q \to p) \to p)$

(c) $p \wedge q \to \neg(\neg p \vee \neg q)$

⟨2⟩ 4. Design elimination and introduction rules for the propositional connective $\leftrightarrow$ (equivalence), and show that they are derived rules if $\varphi \leftrightarrow \psi$ is interpreted as $(\varphi \to \psi) \wedge (\psi \to \varphi)$.