universität innsbruck

Logik	SS 2024	LVA 703026 + 703027
Week 4		April 11, 2024

Selected Solutions

1 (b) Under the valuation $v(p) = v(q) = \mathsf{T}$ and $v(s) = v(t) = \mathsf{F}$, the formulas $p \to q$ and $s \to t$ evaluate to T whereas $p \vee s \to q \wedge t$ evaluates to F. Hence $p \to q, s \to t \not\models p \vee s \to q \wedge t$. From the soundness of natural deduction it follows that the sequent $p \to q, s \to t \vdash p \vee s \to q \wedge t$ is not valid.

2 (a) The following refutation shows that the given clausal form is unsatisfiable:

1. $\{\neg p, \neg q\}$
2. $\{p, \neg q, r\}$
3. $\{p, \neg s\}$
4. $\{q\}$
5. $\{\neg r, s\}$
6. $\{p, \neg r\}$ resolve 3, 5, s
7. $\{p, \neg q\}$ resolve 2, 6, r
8. $\{\neg q\}$ resolve 1, 7, p
9. $\square$ resolve 4, 8, q

(b) Resolution produces the following clauses:

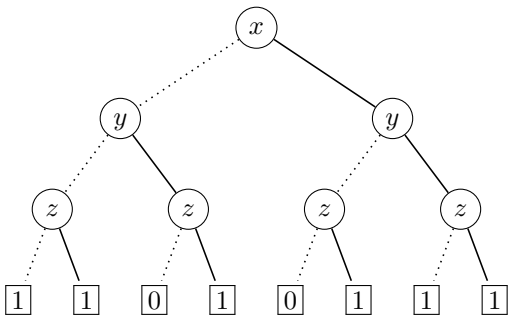
1.	$\{p, q, r\}$	
2.	$\{p, \neg q, r\}$	
3.	$\{q, \neg r\}$	
4.	$\{\neg p, r\}$	
5.	$\{p, r\}$	resolve 1, 2, q
6.	$\{p, q\}$	resolve 1, 3, r
7.	$\{q, r\}$	resolve 1, 4, p
8.	$\{p, r, \neg r\}$	resolve 2, 3, q
9.	$\{p, q, \neg q\}$	resolve 2, 3, r
10.	$\{\neg q, r\}$	resolve 2, 4, p
11.	$\{\neg p, q\}$	resolve 3, 4, r
12.	$\{r, \neg r\}$	resolve 3, 10, q
13.	$\{q, \neg q\}$	resolve 3, 10, r
14.	$\{q\}$	resolve 6, 11, p
15.	$\{r\}$	resolve 7, 10, q
16.	$\{p, q, \neg r\}$	resolve 3, 8, r
17.	$\{p, \neg p, r\}$	resolve 4, 8, r
18.	$\{q, \neg q, r\}$	resolve 4, 9, p
19.	$\{p, \neg p, q\}$	resolve 9, 11, q
20.	$\{q, r, \neg r\}$	resolve 8, 11, p
21.	$\{\neg p, q, r\}$	resolve 11, 17, p
22.	$\{p, q, \neg q, r\}$	resolve 9, 17, p
23.	$\{p, q, r, \neg r\}$	resolve 16, 17, p
24.	$\{p, \neg p, q, r\}$	resolve 17, 19, p

No further resolvents exist and hence φ is satisfiable.

3 (a) From the truth table

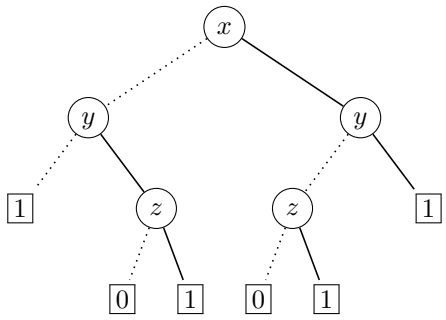
x	y	z	$f(x, y, z)$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

we obtain the binary decision tree

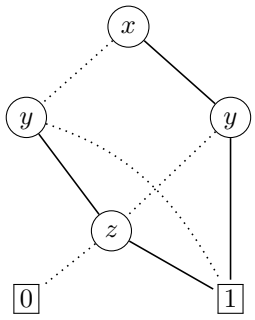


This tree is an ordered BBD (with variable order $[x, y, z]$) but is not reduced because transformations C1 and C2 are applicable.

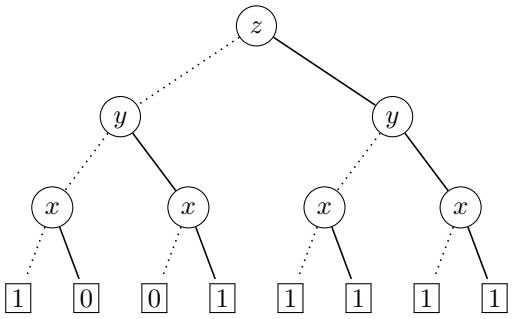
(b) Using transformation C2 on the binary decision tree of part (a) produces



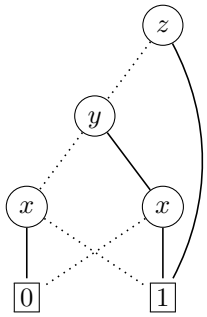
An application of transformation C1 followed by an application of transformation C3 produces the reduced OBDD



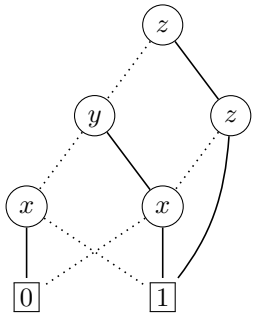
To obtain a second one, we must change the variable order. Let's choose $[z, y, x]$. The resulting binary decision tree



is transformed into the reduced OBDD



(c) For instance, by inserting a redundant test in the second reduced OBDD of part (b)



we obtain a BDD that is reduced but not ordered since there are two z nodes on the same path.