Logik | SS 2024 | LVA 703026 + 703027

Week 5 | April 18, 2024

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. Solutions for bonus exercises must be submitted separately. The (strict) deadline is 7 am on April 18.

Exercises

⟨2⟩ 1. (a) Use resolution to determine whether the formula

$$(p \to q) \land (r \to \neg s) \land ((\neg p \land \neg r) \to \neg s) \land \neg(\neg s \lor \neg(q \to r))$$

is satisfiable.

(b) Use resolution to determine whether the formula

$$((p \to (p \to p)) \to p) \to (\neg p \to \bot)$$

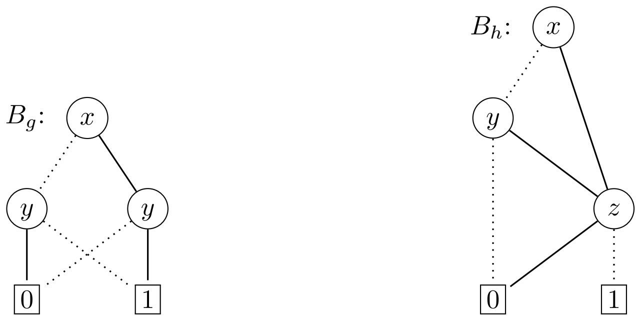
is valid.

⟨2⟩ 2. Is the sequent

$$p \land q \to r \vdash (p \to r) \lor (q \to r)$$

valid? Either give a natural deduction proof or find a model which does not satisfy it.

⟨3⟩ 3. Consider the boolean function f defined by $f(x, y, z) = x(y \oplus z) \oplus (x + \overline{z})$ and the following two reduced OBDDs:



(a) Construct a reduced OBDD for f with variable ordering $[x, y, z]$.

(b) Compute $\mathsf{apply}(\oplus, B_g, B_h)$.

(c) Starting from B_h, compute a reduced OBDD that is equivalent to $\forall y.h$.

⟨3⟩ 4. Consider the predicate logic formula $\varphi = \neg\forall x\, (\exists y\, P(f(x), y, z) \to \forall z\, Q(x, g(y, z)))$. Here f is a unary function symbol, g is a binary function symbol, P is a binary predicate symbol and Q is a ternary predicate symbol.

(a) Draw the parse tree of φ and list all its subformulas.

(b) Which variable occurrences are bound? Which are free?

(c) For each of the following terms t, compute $\varphi[t/x]$, $\varphi[t/y]$, and $\varphi[t/z]$. Is t free for $x/y/z$ in φ?

i. $f(z)$

ii. $g(y, x)$

iii. $g(f(y), y)$

Bonus Exercise

⟨2⟩ 5. (a) Prove the soundness of resolution: If a set S of clauses admits a refutation then S is unsatisfiable.

⟨3⟩ (b) Show that the runtime of resolution may be exponential in its input size. More precisely, let the size of a clause be the number of its literals and let the size of a clausal form be the sum of the sizes of its clauses. Find clausal forms ψ_n whose size grows linearly with n, such that resolution of ψ_n will add an exponential (in n) number of resolvents.