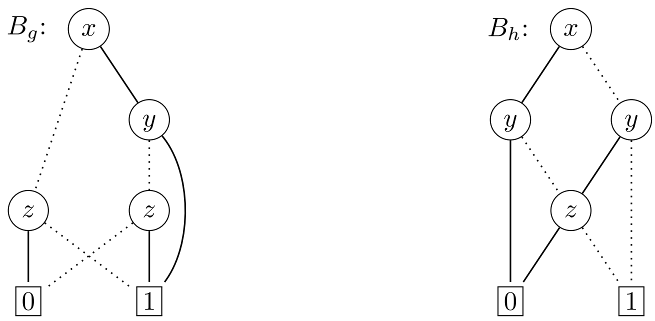
universität innsbruck

Logik	SS 2024	LVA 703026 + 703027
Week 6		April 25, 2024

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. The (strict) deadline is 7 am on April 25.

Exercises

⟨2⟩ 1. Consider the boolean function f defined by $f(x, y, z) = \overline{x}\,\overline{y} \oplus \overline{x}z \oplus \overline{y}z$ and the following two reduced OBDDs:



   (a) Construct a reduced OBDD for f with variable order $[y, z, x]$.

   (b) Compute $\mathsf{apply}(+, B_g, B_h)$.

⟨2⟩ 2. Consider the formula

$$\varphi \;=\; \forall x\, \forall y\, P(f(x, y), f(y, y), z)$$

Here P is a ternary predicate symbol and f is a binary function symbol. Construct two models $\mathcal{M}$ and $\mathcal{M}'$ with respective environments l and l' such that $\mathcal{M} \vDash_l \varphi$ and $\mathcal{M}' \nvDash_{l'} \varphi$.

⟨3⟩ 3. Which of the following sets of sentences are consistent? Prove your answer.

   (a) $\{\forall x\, \exists y\, P(x, y), \exists y\, \forall x\, \neg P(x, y), \forall x\, \neg P(x, x)\}$

   (b) $\{\forall x\, (\exists y\, P(x, y) \to \neg P(x, x)), \exists x\, \forall y\, P(y, x), \neg\forall x\, \neg P(x, x)\}$

⟨3⟩ 4. For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

   (a) $\vdash\; \forall x\, \exists y\, (P(x) \to Q(y)) \to \forall x\, (P(x) \to \exists y\, Q(y))$

   (b) $\forall x\, P(f(x), x) \;\vdash\; \exists y\, \forall x\, P(y, x)$

   (c) $\exists x\, \exists y\, (P(x, y) \vee P(y, x)),\ \neg\exists x\, P(x, x) \;\vdash\; \exists x\, \exists y\, \neg(x = y)$