Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. The (strict) deadline is 7 am on May 2.

Exercises

⟨3⟩ 1. For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

(a) $\exists x\, P(x) \to Q \vdash \forall x\, (P(x) \to Q)$

(b) $\vdash \forall x\, \forall y\, (P(x,y) \to \neg\forall z\, (P(x,z) \to \neg P(z,y)))$

(c) $\forall x\, (P(x) \to Q) \vdash \exists x\, P(x) \to Q$

⟨2⟩ 2. Determine the validity of the following formula:

$$\exists x\, (D(x) \to \forall y\, D(y))$$

This formula models the Drinker Paradox: *There is someone in the pub such that, if he or she is drinking, then everyone in the pub is drinking.*

⟨3⟩ 3. Which of the following pairs of atomic formulas are unifiable? For those that are, compute a most general unifier. Here a and b are constants, and x, y and z are variables.

(a) $P(f(a,x), g(y,b))$ and $P(f(x,y), z)$

(b) $P(x, f(y,a))$ and $P(y, x)$

(c) $R(h(x, y, g(a,z)))$ and $R(h(g(y,y), g(z,z), g(a,a)))$

⟨2⟩ 4. Transform the formula

$$\neg\big(\forall x\, \exists y\, P(f(x,y), y) \to \forall z\, Q(z)\big) \vee \forall x\, R(x,x)$$

into equisatisfiable Skolem normal form.