Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. Solutions for bonus exercises must be submitted separately. The (strict) deadline is 7 am on May 16.

Exercises

⟨3⟩ 1. (a) Using the unification algorithm, determine if the terms

$$f(x, g(y), h(z, g(z))) \quad \text{and} \quad f(h(a, y), g(g(a)), x)$$

are unifiable. If they are unifiable, find a most general unifier. Here a is a constant and x, y and z are variables.

(b) Transform the following sentence into an equisatisfiable Skolem normal form:

$$\varphi = (\forall x\, \exists y\, P(f(x, y), x) \wedge \neg \forall z\, Q(z)) \rightarrow \forall x\, \neg \forall y\, R(y, x)$$

⟨3⟩ 2. Use resolution to determine satisfiability of the following clausal forms, where a is a constant and x and y are variables.

(a) $\{\{P(a), Q(f(a))\}, \{\neg P(x), R(f(x))\}, \{Q(x), \neg R(x)\}\}$

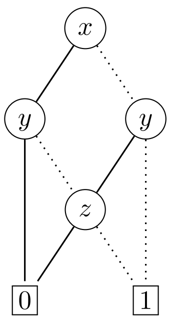
(b) $\{\{P(x), P(f(a))\}, \{\neg P(y), \neg Q(f(x), y)\}, \{Q(f(a), a)\}, \{\neg Q(x, y), Q(f(y), x)\}\}$

⟨3⟩ 3. Compute the algebraic normal forms of the three boolean functions f_1, f_2 and f_3 defined as follows:

$$f_i(x_1, x_2, x_3) = \begin{cases} x_1 & \text{if } s = 0 \\ x_i & \text{if } s = 1 \\ x_s & \text{if } s > 1 \end{cases}$$

for $i \in \{1, 2, 3\}$. Here $s = x_1 + x_2 + x_3$ is the sum of the inputs, which evaluates to a natural number between 0 and 3.

⟨1⟩ 4. Compute the algebraic normal form of the function f represented by the BDD



Bonus Exercise

⟨5⟩ 5. Use a SAT solver to find a 4-coloring for the McGregor map[1] displayed on the next page.

[1] See https://www.cs.cmu.edu/~bryant/boolean/macgregor.html.