universität innsbruck

Logik	SS 2024	LVA 703026 + 703027
Week 10 + 11		June 6, 2024

Selected Solutions

2 **(b)** Since every state satisfies p, no path satisfies $\mathsf{X}\,\neg p$. Hence the paths that satisfy ψ are precisely the paths that satisfy $\mathsf{X}\,q$. All such paths must begin with $1 \to 3$ or $2 \to 3$. Since from state 1 there is also the step to state 2, only state 2 satisfies ψ.

3

1		$\forall x\,(P(x) \vee Q(x))$	premise
2		$\exists x\,\neg Q(x)$	premise
3		$\forall x\,(R(x) \to \neg P(x))$	premise
4	x_0	$\neg Q(x_0)$	assumption
5		$P(x_0) \vee Q(x_0)$	$\forall$e 1
6		$P(x_0)$	assumption
7		$Q(x_0)$	assumption
8		$\bot$	$\neg$e 7, 4
9		$P(x_0)$	$\bot$e 8
10		$P(x_0)$	$\vee$e 5, 6, 7 − 9
11		$R(x_0) \to \neg P(x_0)$	$\forall$e 3
12		$\neg\neg P(x_0)$	$\neg\neg$i 10
13		$\neg R(x_0)$	MT 11, 12
14		$\exists x\,\neg R(x)$	$\exists$i 13
15		$\exists x\,\neg R(x)$	$\exists$e 2, 4 − 14

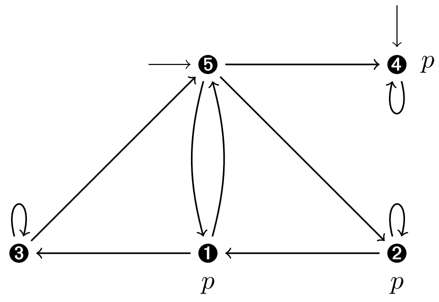
4 **(a)** First we compute the closure $\mathcal{C}(\chi) = \{\chi, \neg\chi, p, \neg p, \mathsf{X}\,p, \neg\,\mathsf{X}\,p\}$. The subsets of $\mathcal{C}(\chi)$ that satisfy the conditions for states of $A_{\neg\chi}$ are

❶ $\{\chi, p, \neg\,\mathsf{X}\,p\}$
❷ $\{\chi, p, \mathsf{X}\,p\}$
❸ $\{\chi, \neg p, \neg\,\mathsf{X}\,p\}$
❹ $\{\neg\chi, p, \mathsf{X}\,p\}$
❺ $\{\neg\chi, \neg p, \mathsf{X}\,p\}$

where ❹ and ❺ are initial states because they contain $\neg\chi$. The conditions on the transition relation give rise to the following 10 transitions:

	❶	❷	❸	❹	❺
❶			✓		✓
❷	✓	✓			
❸			✓		✓
❹				✓	
❺	✓	✓		✓	

In a picture:



(b)

i. The trace $\{p\}^\omega$ is accepted because of the path ❹$^\omega$.

ii. The trace $\{p\}\varnothing\{p\}^\omega$ is not accepted because there is no corresponding path in $A_{\neg\chi}$ that starts in an initial state.

iii. The trace $\varnothing\{p\}\varnothing\{p\}^\omega$ is accepted because of the path ❺❶❺❹$^\omega$.