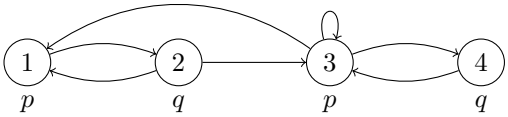
universität innsbruck

Logik	SS 2024	LVA 703026 + 703027
Week 12		June 13, 2024

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. Solutions for bonus exercises must be submitted separately. The (strict) deadline is 7 am on June 13.

Exercises

⟨3⟩ 1. Consider the following model $\mathcal{M}$:



(a) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}$ the CTL formula

$$\varphi \;=\; \mathsf{A}[(p \vee \mathsf{AF}\,\mathsf{EG}\,p)\,\mathsf{U}\,\neg\mathsf{E}[p\,\mathsf{U}\,\mathsf{EX}\,\mathsf{AX}\,q]]$$

holds.

(b) For each $1 \leqslant i < j \leqslant 4$, find an LTL formula $\psi_{i,j}$ that distinguishes states i and j or explain why such a formula does not exist.

⟨2⟩ 2. Are the CTL* formulas $\neg\mathsf{A}[\mathsf{G}\,\mathsf{F}\,\neg p]$ and $\mathsf{E}[\mathsf{F}\,\mathsf{A}[\mathsf{G}\,p]]$ equivalent? Prove your answer.

⟨1⟩ 3. Suppose we extend LTL with a new temporal operator XU:

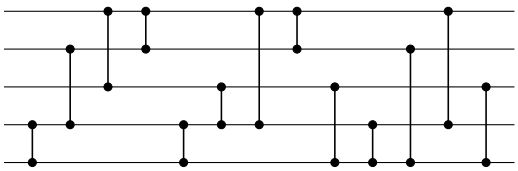
$$\pi \models \varphi\,\mathsf{XU}\,\psi \quad\iff\quad \pi^i \models \psi \text{ for some } i > 1 \text{ and } \pi^j \models \varphi \text{ for all } 1 < j < i$$

Prove that $\{\mathsf{XU}\}$ is an adequate set of temporal connectives for LTL.

⟨2⟩ 4. Use DPLL to determine satisfiability of the CNF

$$\varphi \;=\; (p \vee q \vee r) \wedge (p \vee \neg q \vee r) \wedge (q \vee \neg r) \wedge (\neg q \vee \neg r) \wedge (\neg p \vee s) \wedge (\neg p \vee r \vee \neg s)$$

⟨2⟩ 5. Consider the following comparator network:



(a) Test the network on the input $(5, 1, 2, 4, 3)$.

(b) Determine the depth and size of the network.

(c) Is the network a sorting network?

Bonus Exercise

⟨5⟩ 6. Prove that the CTL* formula $\mathsf{E}[\mathsf{G}\,\mathsf{F}\,p]$ is not expressible in LTL.