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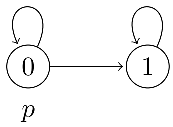
|---|---|---|
| Logik | SS 2024 | LVA 703026 + 703027 |
| Week 12 | | June 13, 2024 |

## Selected Solutions

1 (b) We can take $\psi_{1,2} = \psi_{1,4} = \psi_{2,3} = \psi_{3,4} = p$. We have $\mathcal{M}, 1 \vDash \mathsf{X}\, q$ and $\mathcal{M}, 3 \nvDash \mathsf{X}\, q$. Hence we take $\psi_{1,3} = \mathsf{X}\, q$. The states 2 and 4 cannot be distinguished because they admit exactly the same *traces*: $(\{q\}\{p\}^+)^\omega$ and $(\{q\}\{p\}^+)^*\{q\}\{p\}^\omega$.

2 Consider the model $\mathcal{M}$:



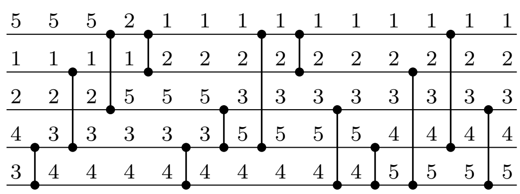

The formula $\neg\mathsf{A}[\mathsf{G}\,\mathsf{F}\,\neg p]$ does hold in state 0 of $\mathcal{M}$ because the path $(0)^\omega$ does not satisfy $\mathsf{G}\,\mathsf{F}\,\neg p$. The formula $\mathsf{E}[\mathsf{F}\,\mathsf{A}[\mathsf{G}\,p]]$ does not hold in state 0 since neither state 0 nor state 1 satisfies $\mathsf{A}[\mathsf{G}\,p]$.

4 The following derivation can be obtained by DPLL:

$$
\begin{array}{lrcll}
 & & \| & \varphi & \\
\Longrightarrow & \overset{d}{r} & \| & \varphi & \text{(decide)} \\
\Longrightarrow & \overset{d}{r}\ q & \| & \varphi & \text{(unit propagate)} \\
\Longrightarrow & \neg r & \| & \varphi & \text{(backtrack)} \\
\Longrightarrow & \neg r\ \overset{d}{p} & \| & \varphi & \text{(decide)} \\
\Longrightarrow & \neg r\ \overset{d}{p}\ s & \| & \varphi & \text{(unit propagate)} \\
\Longrightarrow & \neg r\ \neg p & \| & \varphi & \text{(backtrack)} \\
\Longrightarrow & \neg r\ \neg p\ q & \| & \varphi & \text{(unit propagate)} \\
\Longrightarrow & & \text{fail-state} & & \text{(fail)}
\end{array}
$$

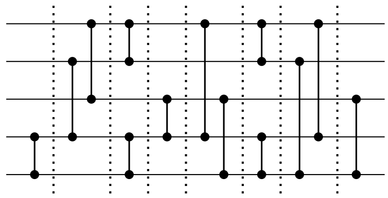
Hence $\varphi$ is unsatisfiable.

5 (a) We have



and so the input $(5, 1, 2, 4, 3)$ is correctly sorted.

(b) The size of the network is 13 and its depth is 8:



(c) No. The input $(1, 3, 2, 4, 5)$ is not sorted: