universität innsbruck

Logik	SS 2024	LVA 703026 + 703027
Week 13		June 20, 2024

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. The (strict) deadline is 7 am on June 20.

Exercises

⟨3⟩ 1. Consider the following CNF φ:

$$(4 \vee \overset{\alpha}{\neg 3} \vee \neg 1) \wedge (\neg 5 \vee \overset{\beta}{\neg 2} \vee \neg 3) \wedge (\overset{\gamma}{\neg 3} \vee 6) \wedge (\neg 4 \vee \overset{\delta}{\neg 6} \vee \neg 7) \wedge (8 \vee \overset{\epsilon}{\neg 6}) \wedge (\neg 4 \vee \overset{\zeta}{\neg 8} \vee 7)$$

   (a) Show that the sequence of decisions 1, 2, 3 followed by applications of unit propagate leads to a conflict, and construct the conflict graph.

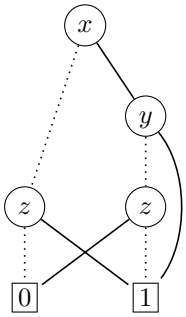
   (b) Starting from the conflict clause in part (a), use resolution to compute the backjump clause corresponding to the first UIP. What is the result of backjump using that clause?

⟨2⟩ 2. Prove the validity of the following sequents using natural deduction.

   (a) $\forall x \, \forall y \, \forall z \, (P(x,z) \vee P(f(y,z),a)) \vdash \forall y \, P(f(y,a),a)$

   (b) $\forall x \, \forall y \, (P(x) \vee P(y)) \vdash \forall x \, \forall y \, (P(x) \wedge P(y))$

⟨2⟩ 3. Consider the boolean function f defined by $f(x,y,z) = \overline{y}(\overline{x} + z) + \overline{x}z$ and the reduced OBDD B_g:



   (a) Show that the set $\{f, \overline{\ }\}$ is not adequate.

   (b) Transform B_g into an equivalent reduced OBDD with variable ordering $[y,z,x]$.

⟨3⟩ 4. For each of the following formulas of predicate logic, determine validity. Either give a natural deduction proof or construct a countermodel.

   (a) $\forall x \, (\exists y \, P(y) \to Q(x)) \to \exists y \, \forall x \, (P(y) \to Q(x))$

   (b) $\forall x \, \exists y \, (P(x) \to Q(y)) \to \forall x \, (P(x) \to \exists y \, Q(y))$

   (c) $\forall x \, \exists y \, (P(y) \to Q(x)) \to \forall x \, (\exists y \, P(y) \to Q(x))$