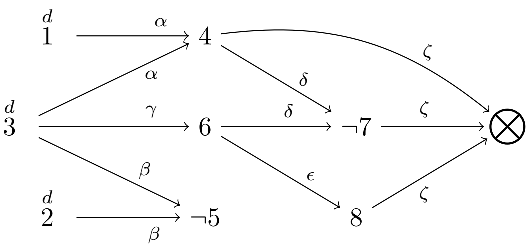
Selected Solutions

1 (a) We obtain the following DPLL derivation:

$$
\begin{array}{lrcll}
 & & \parallel & \varphi & \\
\Longrightarrow & \overset{d}{1} & \parallel & \varphi & (\text{decide}) \\
\Longrightarrow & \overset{d}{1}\ \overset{d}{2} & \parallel & \varphi & (\text{decide}) \\
\Longrightarrow & \overset{d}{1}\ \overset{d}{2}\ \overset{d}{3} & \parallel & \varphi & (\text{decide}) \\
\Longrightarrow & \overset{d}{1}\ \overset{d}{2}\ \overset{d}{3}\ 4 & \parallel & \varphi & (\text{unit propagate}, \alpha) \\
\Longrightarrow & \overset{d}{1}\ \overset{d}{2}\ \overset{d}{3}\ 4\ \neg 5 & \parallel & \varphi & (\text{unit propagate}, \beta) \\
\Longrightarrow & \overset{d}{1}\ \overset{d}{2}\ \overset{d}{3}\ 4\ \neg 5\ 6 & \parallel & \varphi & (\text{unit propagate}, \gamma) \\
\Longrightarrow & \overset{d}{1}\ \overset{d}{2}\ \overset{d}{3}\ 4\ \neg 5\ 6\ \neg 7 & \parallel & \varphi & (\text{unit propagate}, \delta) \\
\Longrightarrow & \overset{d}{1}\ \overset{d}{2}\ \overset{d}{3}\ 4\ \neg 5\ 6\ \neg 7\ 8 & \parallel & \varphi & (\text{unit propagate}, \epsilon)
\end{array}
$$

At this point the clause $\neg 4 \vee \neg 8 \vee 7$ of φ is falsified. The above derivation gives rise to the following implication graph:



We obtain the corresponding conflict graph by removing the nodes $\neg 5$ and $\overset{d}{2}$.

(b) The atoms of the current decision level are 3, 4, 5, 6, 7 and 8. Starting from the conflict clause $\neg 4 \vee \neg 8 \vee 7$, the following clauses are obtained by resolution:

(1)	$\neg 4 \vee \neg 8 \vee 7$	conflict clause
(2)	$\neg 4 \vee \neg 6 \vee 7$	resolving (1) with ϵ
(3)	$\neg 4 \vee \neg 6$	resolving (2) with δ
(4)	$\neg 3 \vee \neg 4$	resolving (3) with γ
(5)	$\neg 1 \vee \neg 3$	resolving (4) with α

The last clause $\neg 1 \vee \neg 3$ contains exactly one literal ($\neg 3$) of the current decision level and thus is a backjump clause. Backjumping according to this clause produces

$$
\begin{array}{lrcll}
 & \overset{d}{1}\ \overset{d}{2}\ \overset{d}{3}\ 4\ \neg 5\ 6\ \neg 7\ 8 & \parallel & \varphi & \\
\Longrightarrow & \overset{d}{1}\ \neg 3 & \parallel & \varphi & (\text{backjump})
\end{array}
$$

or

$$\overset{d}{1}\;\overset{d}{2}\;\overset{d}{3}\;4\;\neg 5\;6\;\neg 7\;8 \quad \| \quad \varphi$$

$$\Longrightarrow \quad \overset{d}{1}\;\overset{d}{2}\;\neg 3 \quad \| \quad \varphi \qquad \text{(backjump)}$$

2 (a)

1	$\forall x\,\forall y\,\forall z\,(P(x,z) \vee P(f(y,z),a))$	premise
2	y_0	
3	$\forall y\,\forall z\,(P(f(y_0,a),z) \vee P(f(y,z),a))$	$\forall$e 1
4	$\forall z\,(P(f(y_0,a),z) \vee P(f(y_0,z),a))$	$\forall$e 3
5	$P(f(y_0,a),a) \vee P(f(y_0,a),a)$	$\forall$e 4
6	$P(f(y_0,a),a)$	assumption
7	$P(f(y_0,a),a)$	assumption
8	$P(f(y_0,a),a)$	$\vee$e 5, 6, 7
9	$\forall y\,P(f(y,a),a)$	$\forall$i 2 – 8

3 (a) From the truth tables

x	y	z	$f(x,y,z)$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

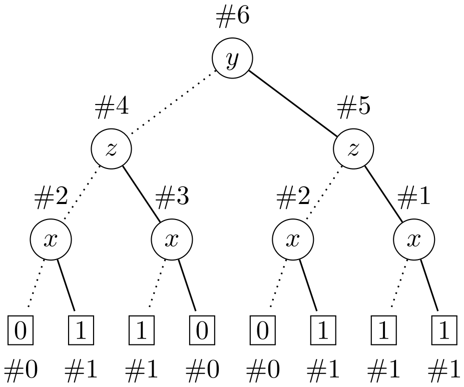
x	$\overline{x}$
0	1
1	0

we infer that both $f(x,y,z)$ and $\overline{x}$ are self-dual. By Post's adequacy theorem it follows that the set $\{f, \overline{}\}$ is not adequate.

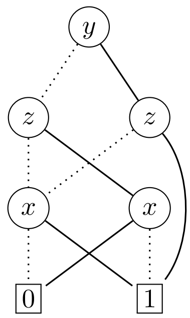
(b) From the truth table

y	z	x	$g(x,y,z)$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

we obtain the binary decision tree



Applying the reduce algorithm produces the desired reduced OBDD:



4 (c) Consider the model $\mathcal{M}$ consisting of the set $\{a, b\}$ together with the interpretations $P^{\mathcal{M}} = \{a\}$ and $Q^{\mathcal{M}} = \varnothing$. Then $\mathcal{M} \vDash \forall x\, \exists y\, (P(y) \to Q(x))$ but not $\mathcal{M} \vDash \forall x\, (\exists y\, P(y) \to Q(x))$. Hence $\mathcal{M} \nvDash \forall x\, \exists y\, (P(y) \to Q(x)) \to \forall x\, (\exists y\, P(y) \to Q(x))$ and we conclude that the given formula is not valid.