SS 2024 lecture 1

Logic

Diana Gründlinger Aart Middeldorp Fabian Mitterwallner
Alexander Montag Johannes Niederhauser Daniel Rainer

Outline

VO is streamed and recorded

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Particify
ars.uibk.ac.at
with session ID **0992 9580** for anonymous questions

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Important Information

- LVA 703026 (VO 3) + 703027 (PS 2)

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Important Information

- LVA 703026 (VO 3) + 703027 (PS 2)
- http://cl-informatik.uibk.ac.at/teaching/ss24/lics

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Important Information

- LVA 703026 (VO 3) + 703027 (PS 2)
- http://cl-informatik.uibk.ac.at/teaching/ss24/lics
- online registration for VO required until June 30

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- LVA 703026 (VO 3) + 703027 (PS 2)
- `http://cl-informatik.uibk.ac.at/teaching/ss24/lics`
- online registration for VO required until June 30
- OLAT links for VO and PS

Time and Place

VO	Monday	8:30 – 11:00	HSB 1	Aart

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TU	Wednesday	16:15 – 17:00	SR 13	Alexander

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VO	Monday	8:30 – 11:00	HSB 1	Aart	
TU	Wednesday	16:15 – 17:00	SR 13	Alexander	
PS	Thursday	12:00 – 13:30	HSB 8	Daniel	group 1
	Thursday	8:30 – 10:00	HSB 4	Fabian	group 2
	Thursday	8:30 – 10:00	SR 12	Johannes	group 3
	Thursday	12:00 – 13:30	HS E	Diana	group 4
	Thursday	13:45 – 15:15	HS 11	Diana	group 5

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	Thursday	13:45 – 15:15	HS 11	Diana	group 5	

PS group change requests until noon tomorrow using SWAp tool

Consultation Hours

Diana Gründlinger	3M03	Thursday	9:30 – 11:00
Aart Middeldorp	3M07	Wednesday	11:30 – 13:00
Fabian Mitterwallner	3M03	Thursday	10:30 – 12:00
Alexander Montag	ÖH Technik	Wednesday	14:00 – 15:00
Johannes Niederhauser	3M03	Friday	9:30 – 11:00
Daniel Rainer	3M03	Wednesday	11:30 – 13:00

Schedule

lecture 1	04.03 & 07.03	lecture 8	06.05 & 16.05
lecture 2	11.03 & 14.03	lecture 9	13.05 & 23.05
lecture 3	18.03 & 21.03	lecture 10	27.05 & 06.06
lecture 4	08.04 & 11.04	lecture 11	03.06 & 06.06
lecture 5	15.04 & 18.04	lecture 12	10.06 & 13.06
lecture 6	22.04 & 25.04	lecture 13	17.06 & 20.06
lecture 7	29.04 & 02.05	lecture 14	24.06

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lecture 4	08.04 & 11.04	lecture 11	03.06 & 06.06
lecture 5	15.04 & 18.04	lecture 12	10.06 & 13.06
lecture 6	22.04 & 25.04	lecture 13	17.06 & 20.06
lecture 7	29.04 & 02.05	lecture 14	24.06 (first exam)

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Announcements

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Announcements

- VO is streamed and recorded
- PS in presence, no PS on June 27

Grading — VO

- first exam on June 24

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- de-registration is possible until 23:59 on June 20

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- registration starts 5 weeks before exam and ends 2 weeks before exam
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- de-registration is possible until 23:59 on June 20
- second exam on September 20
- third exam on February 26, 2025

Grading — PS

$$\text{score} \;=\; \min\left(\tfrac{50}{67}(E+P)+B,\,100\right)$$

Grading — PS

score $= \min\left(\frac{50}{67}(E+P)+B, 100\right)$ E: points for solved exercises (at most 120)

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grade : $[0,50) \to \mathbf{5}$ $[50,63) \to \mathbf{4}$ $[63,75) \to \mathbf{3}$ $[75,88) \to \mathbf{2}$ $[88,100] \to \mathbf{1}$

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Literature

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Logic in Computer Science (second edition)

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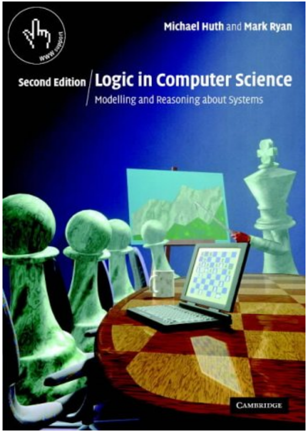

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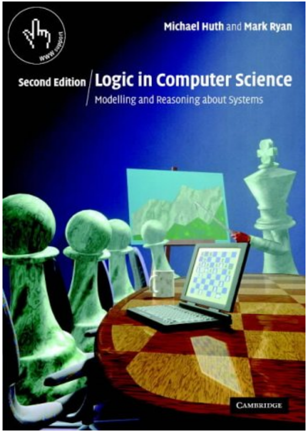

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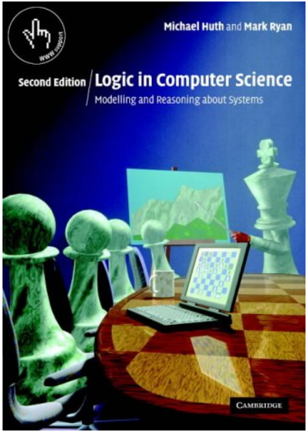

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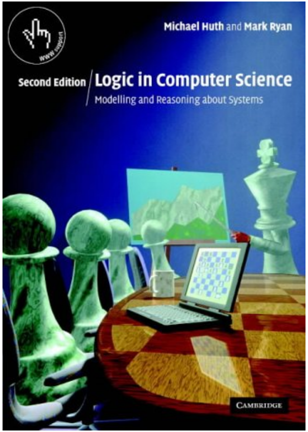

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- slides are available on Thursday before lecture on Monday

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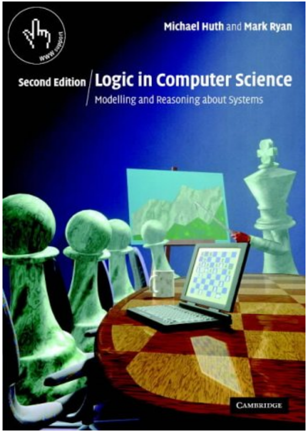

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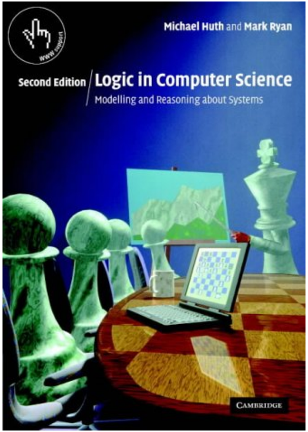

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evaluation SS 2023

Evaluation SS 2023 (selected comments)

- Es macht keinen sinn dass der Professor selbst viele Formeln nicht auswendig kann wir aber alle auswendig wissen müssen für den test. Das hat absolut nichts mit Verständnis zu tun. Es is unnötiges auswendig lernen was sich nichts bringt. In jeder anderen situation würde man z.b. die regeln der natural deduction oder Prädikaten logik nachlesen.

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- The lecture was interesting, but the slides are very poorly structured or very often something is assumed but not explained why. It's a lot to memorize without the professor explaining/reciting it in a way that the student understands.

  One of the worst lecturers, much is defined as it is, without explanation. And if students attend more different PS, then his subject is the most important thing to him, he doesn't take students into consideration.

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- explained an extremely difficult topic very well. the logic course in of itself is complicated and extensive so i can't fault him for that. he knows his stuff for sure.
- This was the best VO in this semester. The slides were good, PS sheets helped a lot.

Outline

1. **Introduction**

   Organisation   Motivation   Contents

2. **Propositional Logic**

3. **Satisfiability and Validity**

4. **Intermezzo**

5. **Conjunctive Normal Forms**

6. **Further Reading**

Logic in Innsbruck

Donnerstag, 11. Mai 2006 — **WISSEN HEUTE** — *LFU INNSBRUCK* **5**

ZEITTAFEL der LFU Innsbruck

1669	Gründung der Universität Innsbruck aus dem seit 100 Jahren bestehenden Jesuitengymnasiums durch Leopold I.
1669/70	Aufnahme des Lehrbetriebs durch die Jesuiten. Erster Universitätskurs wird im Fach Logik abgehalten.
1677	Durch die Bestätigung der Errichtung durch Papst Innozenz XI. erlangt die LFU ihre volle Rechtsgültigkeit. Vor dem Hintergrund wissenschaftlich aufblühender protestantischer Hochschulen sollte Innsbruck das katholische Bollwerk zwischen Deutschland und Italien werden.
1781	Kaiser Joseph I. stufte die Universität Innsbruck zu einem Lyzeum zu Gunsten der Zentraluniversitäten Wien und Prag herab.
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1809	Studentenkompanien beteiligen sich am Tiroler Freiheitskampf.
1810	Aufhebung durch die Bayern

WISSENSWERT

Mit Maria-Theresia kam die Bibliothek an die Universität: Am 22. Mai 1745 genehmigte Maria Theresia die Errichtung einer Innsbrucker Bibliothek. Grundstein bildete die Büchersammlung der Tiroler Habsburger. Die Bibliothek war öffentlich zugänglich und die Benützerordnung war streng: Es durfte immer nur ein Buch vor Ort gelesen werden und auf Bücherentwendungen stand die Exmatrikulation.

Innsbruck zieht Studierende an: 1684 meldet die Chronik des

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Formal Logic at Department of Christian Philosophy

Logic in Computer Science

- During the past 30 years, there has been an extensive and growing interaction between logic and computer science.
- Concepts and methods of logic occupy a central place in computer science, insomuch that logic has been called the calculus of computer science.
- Logic has been much more effective in computer science than it has been in mathematics.

Phokion G. Kolaitis, Moshe Y. Vardi (2001)

Example（数独 Sudoku）

	6		1		4		5	
		8	3		5	6		
2								1
8			4		7			6
		6				3		
7			9		1			4
5								2
		7	2		6	9		
	4		5		8		7	

propositional logic is very useful to quickly develop efficient solver for Sudoku

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	6		1		4		5	
		8	3		5	6		
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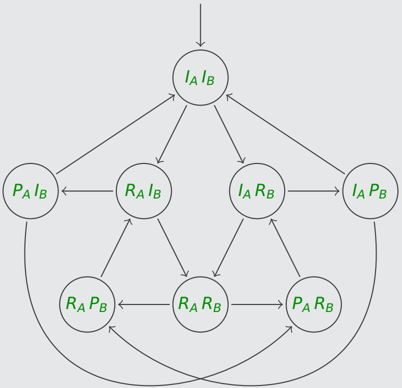
8			4		7			6
		6				3		
7			9		1			4
5								2
		7	2		6	9		
	4		5		8		7	

propositional logic is very useful to quickly develop efficient solver for Sudoku and all kinds of other tasks

Example (Printer Manager)



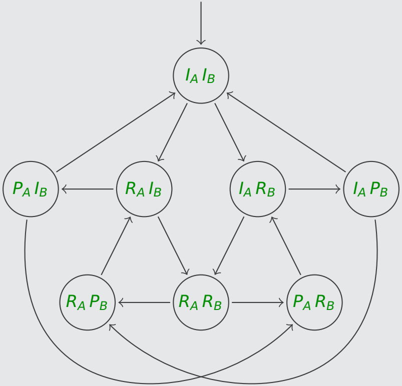
two users A and B

I_i user i is idle

R_i print request by user i

P_i printing document for user i

Example (Printer Manager)



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some questions

- is every P_i preceded by R_i ?

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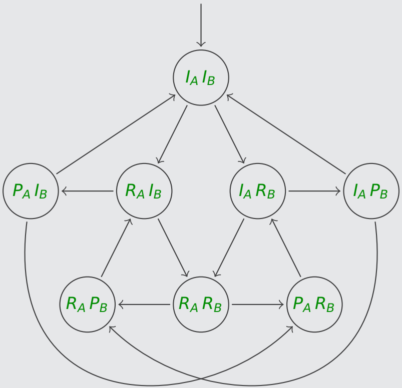

two users A and B

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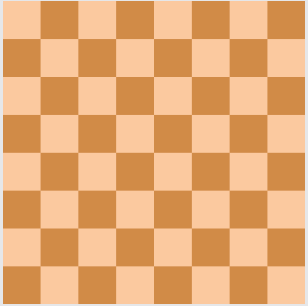
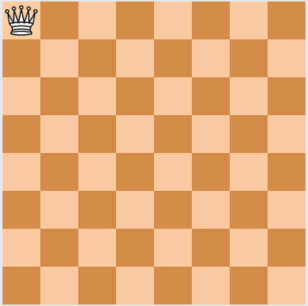
some questions

- is every P_i preceded by R_i ?
- is every R_i eventually followed by P_i ?

Example (Eight Queens Puzzle)



Example (Eight Queens Puzzle)



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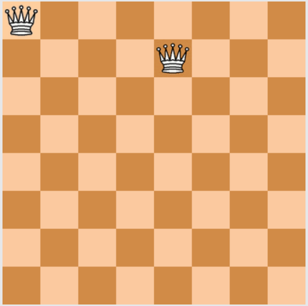

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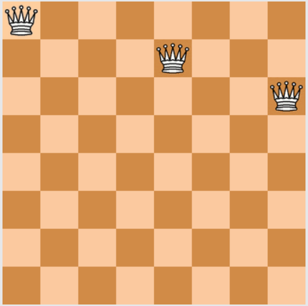

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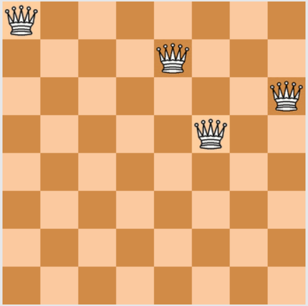

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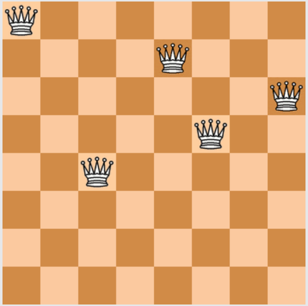

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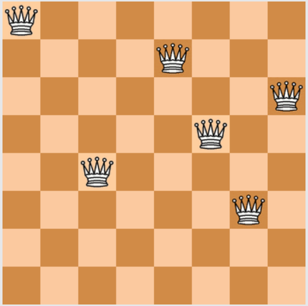

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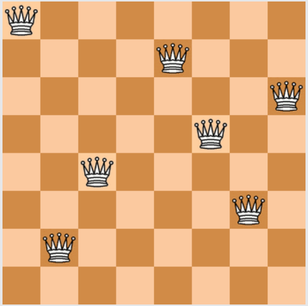

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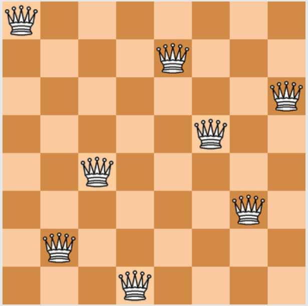

Prolog code

```
:- use_module(library(clpfd)).

nqueens(N,Qs) :-
  length(Qs,N), Qs ins 1 .. N,
  all_different(Qs),
  constraint_queens(Qs), label(Qs).

constraint_queens([]).
constraint_queens([Q|Qs]) :-
  noattack(Q,Qs,1),
  constraint_queens(Qs).

noattack(_,[],_).
noattack(X,[Q|Qs],N) :-
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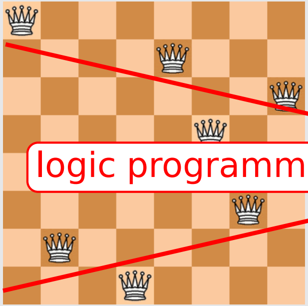

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```
?- nqueens(8,Xs).
```

logic programming is sometimes taught in elective module

Example (Coloring Austria)

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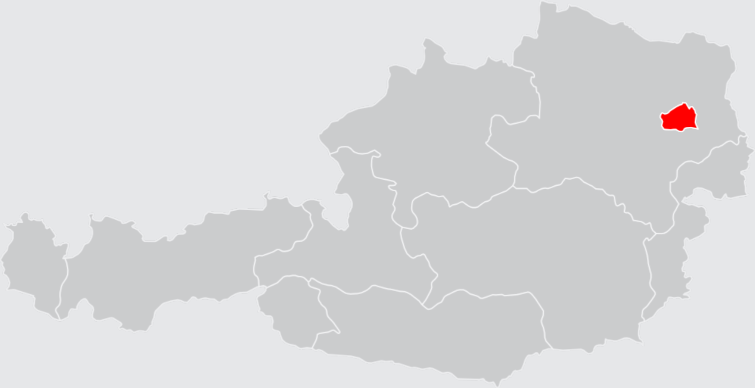

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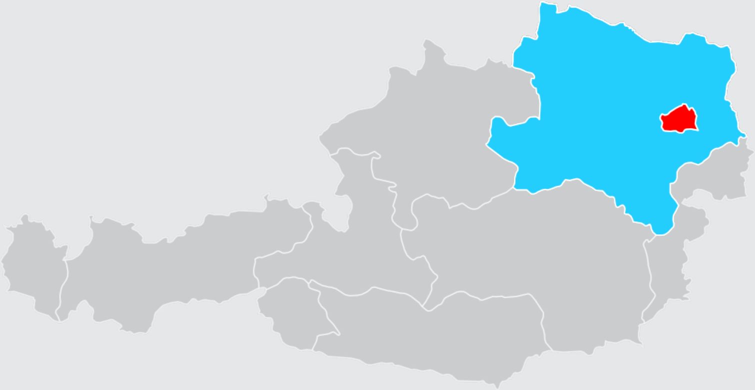

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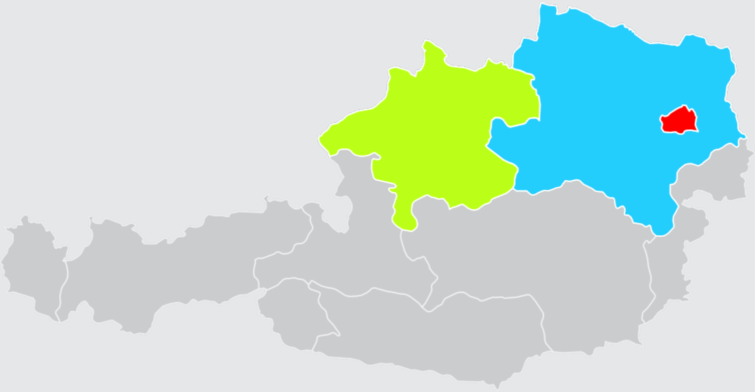

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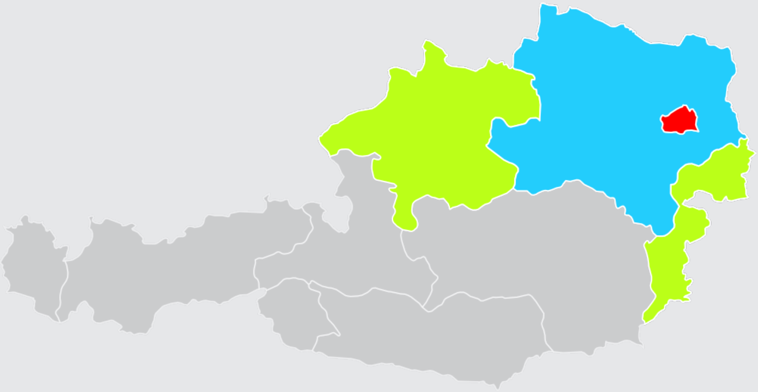

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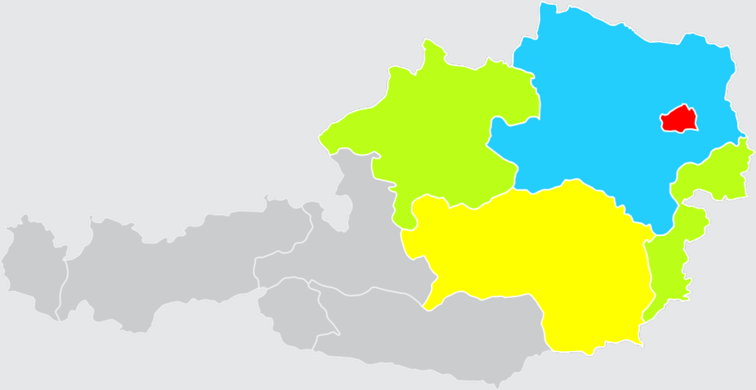

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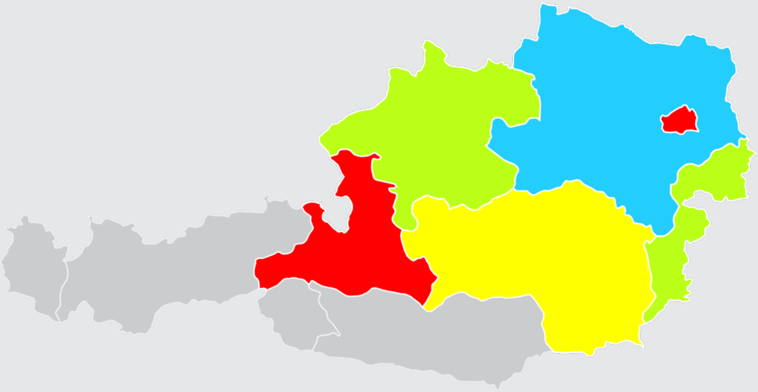

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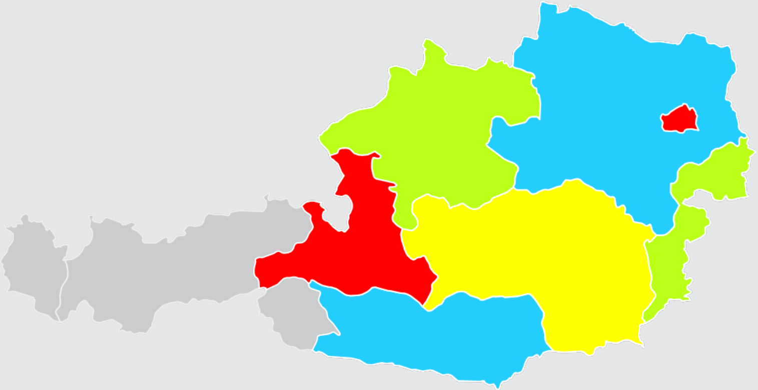

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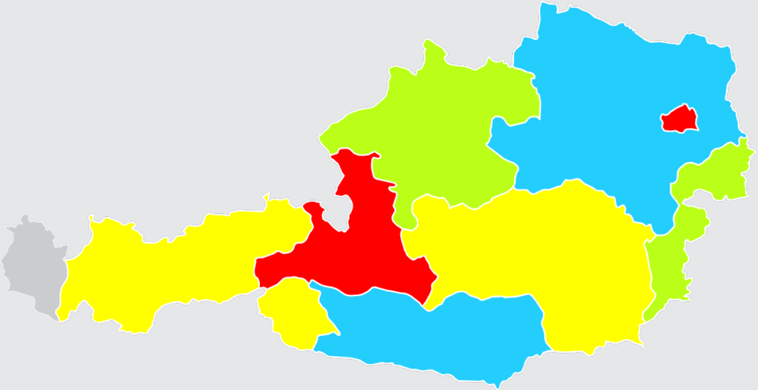

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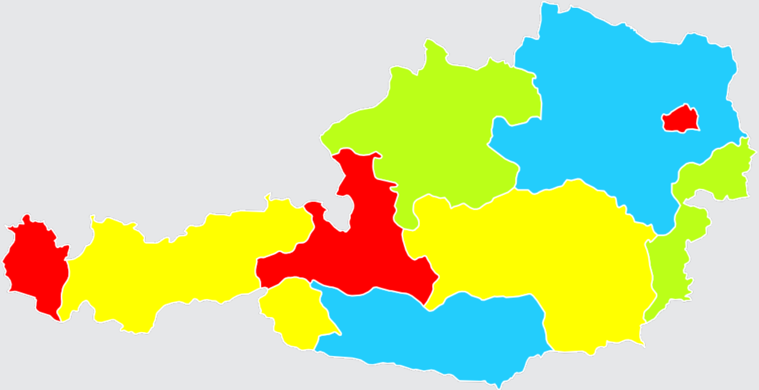

question: do three colors suffice to color Austria ?

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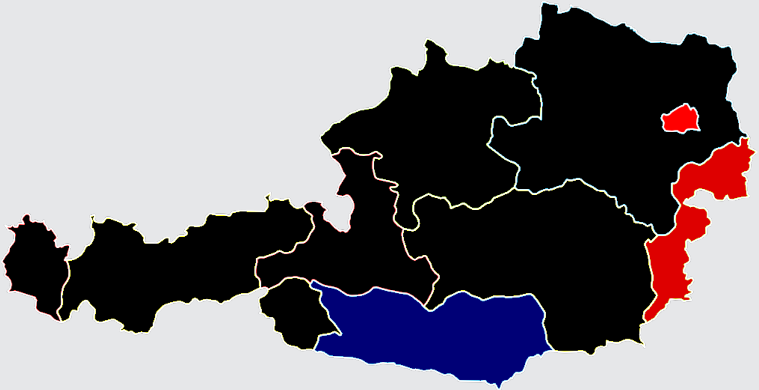

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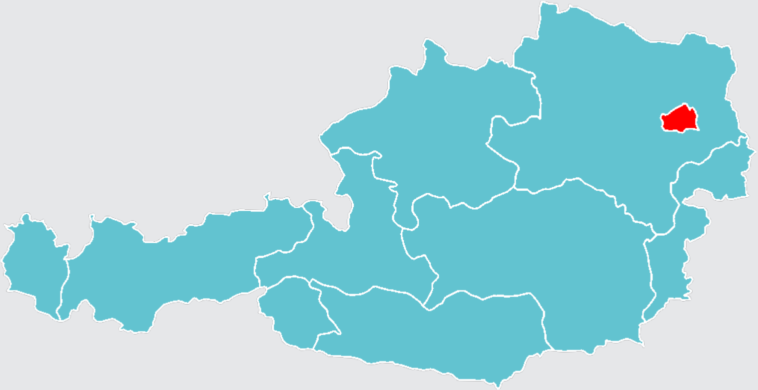

question: do two colors suffice to color Austria ?!

Example (Programming Task)

function that computes $\sum_{i=1}^{n} i$ (exercise in C programming course at Nagoya University)

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some answers:

```
int sum(int x) {
  int i, j, z;
  z = 0;
  for (i = 0; i <= x; i++)
    for (j = 0; j < i; j++)
      z++;
  return z;
}
```

```
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  if (n <= 0) {
    return 0;
  } else {
    return (n*(n+1)/2);
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question: are these programs correct ?

Greek Alphabet

alpha	α	A	eta	η	H	nu	ν	N	tau	τ	T
beta	β	B	theta	θ ϑ	Θ	xi	ξ	Ξ	upsilon	υ	Υ
gamma	γ	Γ	iota	ι	I	omicron	o	O	phi	ϕ φ	Φ
delta	δ	Δ	kappa	κ	K	pi	π	Π	chi	χ	X
epsilon	ϵ ε	E	lambda	λ	Λ	rho	ρ	P	psi	ψ	Ψ
zeta	ζ	Z	mu	μ	M	sigma	σ ς	Σ	omega	ω	Ω

Outline

Part I: Propositional Logic

algebraic normal forms, binary decision diagrams, conjunctive normal forms, DPLL, Horn formulas, natural deduction, Post's adequacy theorem, resolution, SAT, semantics, sorting networks, soundness and completeness, syntax, Tseitin's transformation

Part II: Predicate Logic

natural deduction, quantifier equivalences, resolution, semantics, Skolemization, syntax, undecidability, unification

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propositional formulas are built from

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top	$\top$		
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according to following Backus–Naur Form:

$$\varphi ::= p \mid \bot \mid \top \mid (\neg\varphi) \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid (\varphi \rightarrow \varphi)$$

Notational Conventions

- binding precedence $\neg \quad > \quad \wedge, \vee \quad > \quad \rightarrow$

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- $\rightarrow$, $\wedge$, $\vee$ are right-associative: $p \rightarrow q \rightarrow r$ denotes $p \rightarrow (q \rightarrow r)$

Example

formula $\neg(\neg(p \vee (q \wedge \neg p)) \rightarrow r)$

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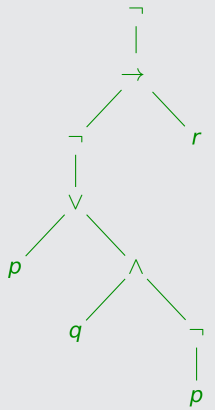
parse tree



Example

formula $\neg(\neg(p \vee (q \wedge \neg p)) \to r)$ $\qquad$ $\neg\neg(p \vee q) \wedge (\neg p \to r)$

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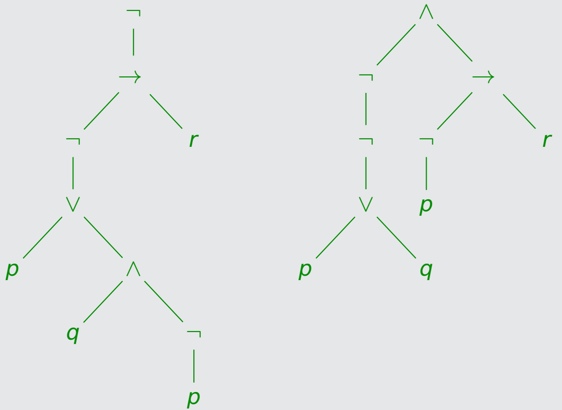

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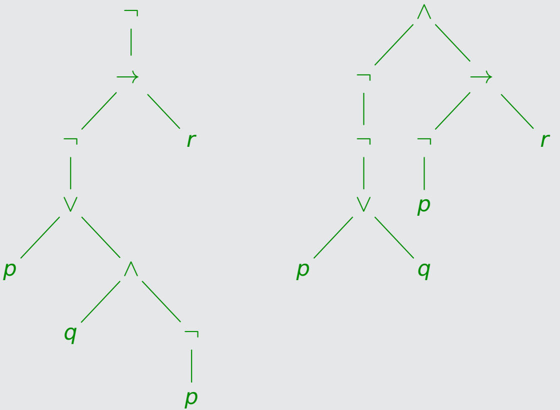

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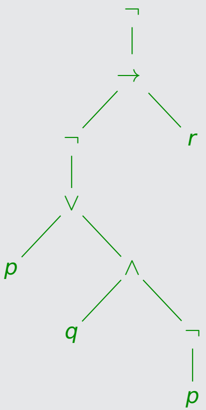
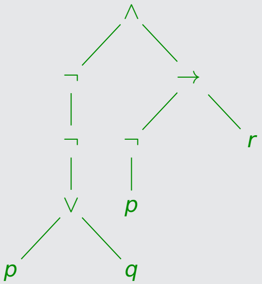
parse tree			?

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  truth values

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Example

$v(p) = T$ and $v(q) = F \quad \Longrightarrow \quad \bar{v}(p \wedge \neg q \to \neg p) = F$

Definition

truth tables

φ	$\neg\varphi$
T	F
F	T

φ	ψ	$\varphi \wedge \psi$	$\varphi \vee \psi$	$\varphi \rightarrow \psi$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
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T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

truth tables for propositional formulas are constructed bottom-up

Example ❶

p	q	$(p \rightarrow \neg q) \rightarrow (q \vee \neg p)$
T	T	
T	F	
F	T	
F	F	

Example ➊

p	q	$\neg p$	$(p \rightarrow \neg q) \rightarrow (q \vee \neg p)$
T	T	F	
T	F	F	
F	T	T	
F	F	T	

Example ❶

p	q	$\neg p$	$\neg q$	$(p \to \neg q) \to (q \vee \neg p)$
T	T	F	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	

Example ①

p	q	$\neg p$	$\neg q$	$p \rightarrow \neg q$	$(p \rightarrow \neg q) \rightarrow (q \vee \neg p)$
T	T	F	F	F	
T	F	F	T	T	
F	T	T	F	T	
F	F	T	T	T	

Example ❶

p	q	$\neg p$	$\neg q$	$p \rightarrow \neg q$	$q \vee \neg p$	$(p \rightarrow \neg q) \rightarrow (q \vee \neg p)$
T	T	F	F	F	T	
T	F	F	T	T	F	
F	T	T	F	T	T	
F	F	T	T	T	T	

Example ❶

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T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Example ❶

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T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Example ❷

p	q	$(p \to \neg q) \to (q \vee \neg p)$
T	T	
T	F	
F	T	
F	F	

Example ①

p	q	$\neg p$	$\neg q$	$p \to \neg q$	$q \vee \neg p$	$(p \to \neg q) \to (q \vee \neg p)$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Example ②

p	q	$(p \to \neg q) \to (q \vee \neg p)$		
T	T			T
T	F			
F	T	T		T
F	F	T		

Example ❶

p	q	$\neg p$	$\neg q$	$p \to \neg q$	$q \lor \neg p$	$(p \to \neg q) \to (q \lor \neg p)$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Example ❷

p	q	$(p \to \neg q)$	$\to$	$(q \lor \neg p)$
T	T		T	T
T	F			
F	T	T	T	T
F	F	T		

Example ❶

p	q	$\neg p$	$\neg q$	$p \to \neg q$	$q \vee \neg p$	$(p \to \neg q) \to (q \vee \neg p)$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Example ❷

p	q	$(p$	$\to$	$\neg q)$	$\to$	$(q$	$\vee$	$\neg p)$
T	T				T		T	
T	F			T				
F	T		T		T		T	
F	F		T					

Example ❶

p	q	$\neg p$	$\neg q$	$p \to \neg q$	$q \vee \neg p$	$(p \to \neg q) \to (q \vee \neg p)$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Example ❷

p	q	$(p$	$\to$	$\neg q)$	$\to$	$(q$	$\vee$	$\neg p)$
T	T				T		T	
T	F		T	T				
F	T		T		T		T	
F	F		T					

Example ❶

p	q	$\neg p$	$\neg q$	$p \rightarrow \neg q$	$q \vee \neg p$	$(p \rightarrow \neg q) \rightarrow (q \vee \neg p)$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Example ❷

p	q	$(p$	$\rightarrow$	$\neg q)$	$\rightarrow$	$(q$	$\vee$	$\neg p)$
T	T				T		T	
T	F		T	T				F
F	T		T		T		T	
F	F		T					T

Example ❶

p	q	$\neg p$	$\neg q$	$p \to \neg q$	$q \vee \neg p$	$(p \to \neg q) \to (q \vee \neg p)$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Example ❷

p	q	$(p$	$\to$	$\neg q)$	$\to$	$(q$	$\vee$	$\neg p)$
T	T				T		T	
T	F		T	T			F	F
F	T		T		T		T	
F	F		T				T	T

Example ❶

p	q	$\neg p$	$\neg q$	$p \to \neg q$	$q \vee \neg p$	$(p \to \neg q) \to (q \vee \neg p)$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Example ❷

p	q	$(p \to$	$\neg q)$	$\to$	$(q \vee$	$\neg p)$
T	T			**T**	T	
T	F	T	T	**F**	F	F
F	T	T		**T**	T	
F	F	T		**T**	T	T

Definitions

- semantic entailment

$$\varphi_1, \varphi_2, \ldots, \varphi_n \vDash \psi$$

if $\bar{v}(\psi) = \mathsf{T}$ whenever $\bar{v}(\varphi_1) = \bar{v}(\varphi_2) = \cdots = \bar{v}(\varphi_n) = \mathsf{T}$ for every valuation v

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Examples ❶

p	q	$p \to q \vDash \neg p \vee q$
T	T	T
T	F	F
F	T	T
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Examples ❶

p	q	$p \to q$ $\vDash$	$\neg p \vee q$
T	T	T	T
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F	T	T	T
F	F	T	T

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- tautology is formula φ such that $\vDash \varphi$

Examples ①

p	q	$p \rightarrow q$	$\vDash$	$\neg p \vee q$
T	T	T		T
T	F	F		
F	T	T		T
F	F	T		T

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Examples ❶

p	q	$p \to q$	$\vDash$	$\neg p \vee q$
T	T	T		T
T	F	F		
F	T	T		T
F	F	T		T

p	q	$p \to q,$	$p \to \neg q$	$\vDash$	$\neg p$
T	T	T			
T	F	F			
F	T	T			
F	F	T			

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Examples ①

p	q	$p \to q$	$\vDash$	$\neg p \vee q$
T	T	T		T
T	F	F		
F	T	T		T
F	F	T		T

p	q	$p \to q$,	$p \to \neg q$	$\vDash$	$\neg p$
T	T	T	F		
T	F	F			
F	T	T	T		
F	F	T	T		

Definitions

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Examples ①

p	q	$p \to q$	$\vDash$	$\neg p \vee q$
T	T	T		T
T	F	F		
F	T	T		T
F	F	T		T

p	q	$p \to q$,	$p \to \neg q$	$\vDash$	$\neg p$
T	T	T	F		
T	F	F			
F	T	T	T		T
F	F	T	T		T

Examples ❷

p	q	$p \to q \vDash q \to p$
T	T	T
T	F	F
F	T	T
F	F	T

Examples ❷

p	q	$p \to q$	$\vDash$	$q \to p$
T	T	T		T
T	F	F		
F	T	T		
F	F	T		

Examples ❷

p	q	$p \to q$	$\not\models$	$q \to p$
T	T	T		T
T	F	F		
F	T	T		F
F	F	T		

Examples ❷

p	q	$p \to q$	$\not\models$	$q \to p$
T	T	T		T
T	F	F		
F	T	T		F
F	F	T		

p	q	$p \to q,\ p \to \neg q$	$\models$	q
T	T	T		
T	F	F		
F	T	T		
F	F	T		

Examples ❷

p	q	$p \to q$	$\not\models$	$q \to p$
T	T	T		T
T	F	F		
F	T	T		F
F	F	T		

p	q	$p \to q$,	$p \to \neg q$	$\models q$
T	T	T	F	
T	F	F		
F	T	T	T	
F	F	T	T	

Examples ❷

p	q	$p \rightarrow q$	$\not\models$	$q \rightarrow p$
T	T	T		T
T	F	F		
F	T	T		F
F	F	T		

p	q	$p \rightarrow q,$	$p \rightarrow \neg q$	$\models$	q
T	T	T	F		
T	F	F			
F	T	T	T		T
F	F	T	T		

Examples ❷

p	q	$p \to q$	$\not\models$	$q \to p$
T	T	T		T
T	F	F		
F	T	T		F
F	F	T		

p	q	$p \to q,$	$p \to \neg q$	$\not\models$	q
T	T	T	F		
T	F	F			
F	T	T	T		T
F	F	T	T		F

Examples ❷

p	q	$p \to q$ $\not\models$	$q \to p$
T	T	T	T
T	F	F	
F	T	T	F
F	F	T	

p	q	$p \to q,$	$p \to \neg q$ $\not\models$	q
T	T	T	F	
T	F	F		
F	T	T	T	T
F	F	T	T	F

p	q	$p \to q,\ p \wedge \neg q \models \bot$
T	T	T
T	F	F
F	T	T
F	F	T

Examples ❷

p	q	$p \to q$	$\not\models$	$q \to p$
T	T	T		T
T	F	F		
F	T	T		F
F	F	T		

p	q	$p \to q,$	$p \to \neg q$	$\not\models$	q
T	T	T	F		
T	F	F			
F	T	T	T		T
F	F	T	T		F

p	q	$p \to q,$	$p \wedge \neg q$	$\models$	$\bot$
T	T	T	F		
T	F	F			
F	T	T	F		
F	F	T	F		

Question

$$\models (\neg p \wedge \neg q) \vee (s \wedge u) \vee (r \wedge w) \vee (\neg t \wedge \neg u) \vee (p \wedge r) \vee (q \wedge s) \vee (p \wedge t) \vee (q \wedge u) \vee (\neg r \wedge \neg s) \vee (t \wedge v) \vee (\neg v \wedge \neg w) \quad ?$$

Question

$$\models (\neg p \wedge \neg q) \vee (s \wedge u) \vee (r \wedge w) \vee (\neg t \wedge \neg u) \vee (p \wedge r) \vee (q \wedge s) \vee (p \wedge t) \vee (q \wedge u) \vee (\neg r \wedge \neg s) \vee (t \wedge v) \vee (\neg v \wedge \neg w) \quad ?$$

... truth table has $2^8 = 256$ rows ...

Question

$$\vDash \begin{array}{l}(\neg p \wedge \neg q) \vee (s \wedge u) \vee (r \wedge w) \vee (\neg t \wedge \neg u) \vee (p \wedge r) \vee (q \wedge s) \\ \vee (p \wedge t) \vee (q \wedge u) \vee (\neg r \wedge \neg s) \vee (t \wedge v) \vee (\neg v \wedge \neg w)\end{array} \quad ?$$

... truth table has $2^8 = 256$ rows ...

	6		1		4		5	
		8	3		5	6		
2								1
8			4		7			6
		6				3		
7			9		1			4
5								2
		7	2		6	9		
	4		5		8		7	

... truth table has $2^{459} > 2^{4 \times 100} = 16^{100} > 10^{100}$ rows ...

Outline

Definitions

formula φ is

- valid if $\bar{v}(\varphi) = \mathsf{T}$ for every valuation v

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formula φ is valid $\iff$ $\neg\varphi$ is unsatisfiable

Proof

$$\begin{aligned} \varphi \text{ is valid} &\iff \bar{v}(\varphi) = \mathsf{T} \quad \text{for every valuation } v \\ &\iff \bar{v}(\neg\varphi) = \mathsf{F} \quad \text{for every valuation } v \end{aligned}$$

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&\iff \bar{v}(\neg\varphi) = \mathsf{T} \quad \text{for no valuation } v
\end{aligned}$$

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\begin{aligned}
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&\iff \neg\varphi \text{ is not satisfiable}
\end{aligned}
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&\iff \bar{v}(\neg\varphi) = \mathsf{T} \quad \text{for no valuation } v \\
&\iff \neg\varphi \text{ is not satisfiable} \iff \neg\varphi \text{ is unsatisfiable}
\end{aligned}
$$

Definition

formulas φ and ψ are semantically equivalent ($\varphi \equiv \psi$) if both $\varphi \vDash \psi$ and $\psi \vDash \varphi$

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Examples

$\neg(\varphi \vee \psi) \equiv \neg\varphi \wedge \neg\psi$ $\qquad$ $\neg\neg\varphi \equiv \varphi$ $\qquad$ $\varphi \vee (\psi \wedge \chi) \equiv (\varphi \vee \psi) \wedge (\varphi \vee \chi)$

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validity and satisfiability are decidable

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construct truth table of φ

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$$\neg(\varphi \vee \psi) \equiv \neg\varphi \wedge \neg\psi \qquad \neg\neg\varphi \equiv \varphi \qquad \varphi \vee (\psi \wedge \chi) \equiv (\varphi \vee \psi) \wedge (\varphi \vee \chi)$$

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Proof

construct truth table of φ and inspect last column:

- φ is valid if and only if all entries are T

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Theorem

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Proof

construct truth table of φ and inspect last column:

- φ is valid if and only if all entries are T
- φ is satisfiable if and only if T entry exists

Outline

Question

Given that one and only one answer is correct, which of the following is true ?

- **A** All of the below.
- **B** None of the below.
- **C** One of the above.
- **D** All of the above.
- **E** None of the above.
- **F** None of the above.

Outline

Definitions

- literal is atom p or negation $\neg p$ of atom

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Theorem

validity of CNFs is efficiently decidable

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Theorem

validity of CNFs is efficiently decidable:

$$\text{CNF } \varphi \text{ is valid} \quad \Longleftrightarrow \quad \text{every clause of } \varphi \text{ contains complementary literals}$$

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- clause is disjunction $\ell_1 \vee \cdots \vee \ell_n$ of literals
- conjunctive normal form (CNF) is conjunction $C_1 \wedge \cdots \wedge C_n$ of clauses
- literals ℓ_1 and ℓ_2 are complementary if $\ell_1 = \neg\ell_2$ or $\neg\ell_1 = \ell_2$

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validity of CNFs is efficiently decidable:

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Examples

1. CNF

$$(p \vee q \vee \neg r) \wedge (\neg p \vee \neg r \vee p) \wedge (\neg q)$$

Examples

1. CNF

$$(p \lor q \lor \neg r) \land (\neg p \lor \neg r \lor p) \land (\neg q)$$

clause

Examples

1. CNF

$$(p \vee q \vee \neg r) \wedge (\neg p \vee \neg r \vee p) \wedge (\neg q)$$

complementary literals

Examples

1. CNF

$$(p \vee q \vee \neg r) \wedge (\neg p \vee \neg r \vee p) \wedge (\neg q)$$

not valid

Examples

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$$(p \vee q \vee \neg r) \wedge (\neg p \vee \neg r \vee p) \wedge (\neg q)$$

not valid   witness: $v(p) = v(q) = \mathsf{F}$ and $v(r) = \mathsf{T}$

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$$(p \vee q \vee \neg r) \wedge (\neg p \vee \neg r \vee p) \wedge (\neg q)$$

not valid witness: $v(p) = v(q) = \mathrm{F}$ and $v(r) = \mathrm{T}$

2. CNF

$$(p \vee q \vee \neg p) \wedge (\neg r \vee \neg p \vee r) \wedge (\neg q \vee q)$$

valid

Examples

1. CNF

$$(p \vee q \vee \neg r) \wedge (\neg p \vee \neg r \vee p) \wedge (\neg q)$$

   not valid   witness: $v(p) = v(q) = \mathsf{F}$ and $v(r) = \mathsf{T}$

2. CNF

$$(p \vee q \vee \neg p) \wedge (\neg r \vee \neg p \vee r) \wedge (\neg q \vee q)$$

   valid

Special Cases

- $\bot$ represents empty clause (no literals)

Examples

1. CNF

$$(p \vee q \vee \neg r) \wedge (\neg p \vee \neg r \vee p) \wedge (\neg q)$$

not valid witness: $v(p) = v(q) = \text{F}$ and $v(r) = \text{T}$

2. CNF

$$(p \vee q \vee \neg p) \wedge (\neg r \vee \neg p \vee r) \wedge (\neg q \vee q)$$

valid

Special Cases

- $\bot$ represents empty clause (no literals)
- $\top$ represents empty CNF (no clauses)

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

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Procedure

① eliminate implications

$$\varphi \rightarrow \psi \quad \xrightarrow{①} \quad \neg\varphi \vee \psi$$

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Procedure

① eliminate implications

② push negations towards atoms and remove double negations

$$
\begin{aligned}
\varphi \to \psi &\overset{①}{\longrightarrow} \neg\varphi \lor \psi \\
\neg(\varphi \land \psi) &\overset{②}{\longrightarrow} \neg\varphi \lor \neg\psi \\
\neg(\varphi \lor \psi) &\overset{②}{\longrightarrow} \neg\varphi \land \neg\psi
\end{aligned}
\qquad\qquad
\neg\neg\varphi \overset{②}{\longrightarrow} \varphi
$$

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Procedure

① eliminate implications

② push negations towards atoms and remove double negations

③ distribute disjunction over conjunction

$$\varphi \to \psi \xrightarrow{①} \neg\varphi \vee \psi \qquad\qquad \neg\neg\varphi \xrightarrow{②} \varphi$$

$$\neg(\varphi \wedge \psi) \xrightarrow{②} \neg\varphi \vee \neg\psi \qquad\qquad \varphi \vee (\psi \wedge \chi) \xrightarrow{③} (\varphi \vee \psi) \wedge (\varphi \vee \chi)$$

$$\neg(\varphi \vee \psi) \xrightarrow{②} \neg\varphi \wedge \neg\psi \qquad\qquad (\varphi \wedge \psi) \vee \chi \xrightarrow{③} (\varphi \vee \chi) \wedge (\psi \vee \chi)$$

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Procedure

① eliminate implications

② push negations towards atoms and remove double negations

③ distribute disjunction over conjunction

$$\varphi \to \psi \xrightarrow{①} \neg\varphi \vee \psi \qquad\qquad \neg\neg\varphi \xrightarrow{②} \varphi$$

$$\neg(\varphi \wedge \psi) \xrightarrow{②} \neg\varphi \vee \neg\psi \qquad\qquad \varphi \vee (\psi \wedge \chi) \xrightarrow{③} (\varphi \vee \psi) \wedge (\varphi \vee \chi)$$

$$\neg(\varphi \vee \psi) \xrightarrow{②} \neg\varphi \wedge \neg\psi \qquad\qquad (\varphi \wedge \psi) \vee \chi \xrightarrow{③} (\varphi \vee \chi) \wedge (\psi \vee \chi)$$

Remark

CNF ψ for formula φ might be exponentially larger

Example (CNFs are not unique)

$$\varphi = \neg(p \vee (q \wedge r))$$

Example (CNFs are not unique)

$$\varphi = \neg(p \vee (q \wedge r))$$

$$\xrightarrow{②} \quad \neg p \wedge \neg(q \wedge r)$$

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$$\xrightarrow{③} \neg((p \vee q) \wedge (p \vee r))$$

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$$\xrightarrow{②} (\neg p \wedge \neg q) \vee \neg(p \vee r)$$

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$$\xrightarrow{②} (\neg p \wedge \neg q) \vee \neg(p \vee r) \xrightarrow{②} (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)$$

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$$\xrightarrow{③} ((\neg p \wedge \neg q) \vee \neg p) \wedge ((\neg p \wedge \neg q) \vee \neg r)$$

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$$\varphi = \neg(p \vee (q \wedge r))$$

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$$\xrightarrow{②} (\neg p \wedge \neg q) \vee \neg(p \vee r) \xrightarrow{②} (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)$$

$$\xrightarrow{③} ((\neg p \wedge \neg q) \vee \neg p) \wedge ((\neg p \wedge \neg q) \vee \neg r)$$

$$\xrightarrow{③} ((\neg p \vee \neg p) \wedge (\neg q \vee \neg p)) \wedge ((\neg p \wedge \neg q) \vee \neg r)$$

$$\xrightarrow{③} ((\neg p \vee \neg p) \wedge (\neg q \vee \neg p)) \wedge ((\neg p \vee \neg r) \wedge (\neg q \vee \neg r))$$

CNFs are not unique, even if rules ①, ②, ③ are applied in order

Procedure (extended)

⓪ simplify formulas with $\bot$ and $\top$

① eliminate implications

② push negations towards atoms and remove double negations

③ distribute disjunction over conjunction

Procedure (extended)

⓪ simplify formulas with $\bot$ and $\top$

① eliminate implications

② push negations towards atoms and remove double negations

③ distribute disjunction over conjunction

$\neg\bot \xrightarrow{⓪} \top$	$\bot \wedge \varphi \xrightarrow{⓪} \bot$	$\bot \vee \varphi \xrightarrow{⓪} \varphi$	$\bot \rightarrow \varphi \xrightarrow{⓪} \top$
$\neg\top \xrightarrow{⓪} \bot$	$\top \wedge \varphi \xrightarrow{⓪} \varphi$	$\top \vee \varphi \xrightarrow{⓪} \top$	$\top \rightarrow \varphi \xrightarrow{⓪} \varphi$
	$\varphi \wedge \bot \xrightarrow{⓪} \bot$	$\varphi \vee \bot \xrightarrow{⓪} \varphi$	$\varphi \rightarrow \bot \xrightarrow{⓪} \neg\varphi$
	$\varphi \wedge \top \xrightarrow{⓪} \varphi$	$\varphi \vee \top \xrightarrow{⓪} \top$	$\varphi \rightarrow \top \xrightarrow{⓪} \top$

Example

$$p \vee (q \wedge (\top \to (\neg p \vee \bot)) \to (\top \wedge \neg q))$$

Example

$$p \vee (q \wedge (\top \rightarrow (\neg p \vee \bot)) \rightarrow (\top \wedge \neg q))$$

$$\xrightarrow{\text{⓪}} \quad p \vee (q \wedge (\top \rightarrow \neg p) \rightarrow (\top \wedge \neg q))$$

Example

$$p \vee (q \wedge (\top \to (\neg p \vee \bot)) \to (\top \wedge \neg q))$$

$$\xrightarrow{⓪} \; p \vee (q \wedge (\top \to \neg p) \to (\top \wedge \neg q)) \; \xrightarrow{⓪} \; p \vee (q \wedge (\top \to \neg p) \to \neg q)$$

Example

$$p \vee (q \wedge (\top \to (\neg p \vee \bot)) \to (\top \wedge \neg q))$$

$$\xrightarrow{⓪} \; p \vee (q \wedge (\top \to \neg p) \to (\top \wedge \neg q)) \; \xrightarrow{⓪} \; p \vee (q \wedge (\top \to \neg p) \to \neg q)$$

$$\xrightarrow{⓪} \; p \vee (q \wedge \neg p \to \neg q)$$

Example

$$p \vee (q \wedge (\top \to (\neg p \vee \bot)) \to (\top \wedge \neg q))$$

$$\xrightarrow{⓪} \; p \vee (q \wedge (\top \to \neg p) \to (\top \wedge \neg q)) \; \xrightarrow{⓪} \; p \vee (q \wedge (\top \to \neg p) \to \neg q)$$

$$\xrightarrow{⓪} \; p \vee (q \wedge \neg p \to \neg q) \; \xrightarrow{①} \; p \vee (\neg (q \wedge \neg p) \vee \neg q)$$

Example

$$p \vee (q \wedge (\top \to (\neg p \vee \bot)) \to (\top \wedge \neg q))$$

$$\xrightarrow{⓪} \ p \vee (q \wedge (\top \to \neg p) \to (\top \wedge \neg q)) \ \xrightarrow{⓪} \ p \vee (q \wedge (\top \to \neg p) \to \neg q)$$

$$\xrightarrow{⓪} \ p \vee (q \wedge \neg p \to \neg q) \ \xrightarrow{①} \ p \vee (\neg(q \wedge \neg p) \vee \neg q)$$

$$\xrightarrow{②} \ p \vee ((\neg q \vee \neg\neg p) \vee \neg q)$$

Example

$$p \vee (q \wedge (\top \to (\neg p \vee \bot)) \to (\top \wedge \neg q))$$

$$\xrightarrow{⓪} p \vee (q \wedge (\top \to \neg p) \to (\top \wedge \neg q)) \xrightarrow{⓪} p \vee (q \wedge (\top \to \neg p) \to \neg q)$$

$$\xrightarrow{⓪} p \vee (q \wedge \neg p \to \neg q) \xrightarrow{①} p \vee (\neg(q \wedge \neg p) \vee \neg q)$$

$$\xrightarrow{②} p \vee ((\neg q \vee \neg\neg p) \vee \neg q) \xrightarrow{②} p \vee ((\neg q \vee p) \vee \neg q)$$

Example (CNF from truth table)

$\varphi = \neg(p \vee (q \wedge r))$

p	q	r	φ
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F

p	q	r	φ
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	T

Example (CNF from truth table)

$\varphi = \neg(p \vee (q \wedge r))$

p	q	r	φ
T	T	T	F
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p	q	r	φ
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	T

$$\varphi \equiv \neg\big((p \wedge q \wedge r) \qquad\qquad\qquad\qquad \big)$$

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p	q	r	φ
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F

p	q	r	φ
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	T

$\varphi \equiv \neg((p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \qquad)$

Example (CNF from truth table)

$\varphi = \neg(p \vee (q \wedge r))$

p	q	r	φ
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F

p	q	r	φ
F	T	T	F
F	T	F	T
F	F	T	T
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$$\varphi \equiv \neg\big((p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \qquad \big)$$

Example (CNF from truth table)

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p	q	r	φ
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T	T	F	F
T	F	T	F
T	F	F	F

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F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	T

$$\varphi \equiv \neg\big((p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \qquad \big)$$

Example (CNF from truth table)

$\varphi = \neg(p \vee (q \wedge r))$

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T	T	T	F
T	T	F	F
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T	F	F	F

p	q	r	φ
F	T	T	F
F	T	F	T
F	F	T	T
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$$\varphi \equiv \neg\big((p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge r)\big)$$

Example (CNF from truth table)

$\varphi = \neg(p \vee (q \wedge r))$

p	q	r	φ	p	q	r	φ
T	T	T	F	F	T	T	F
T	T	F	F	F	T	F	T
T	F	T	F	F	F	T	T
T	F	F	F	F	F	F	T

$$\begin{aligned}\varphi &\equiv \neg\big((p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge r)\big) \\ &\equiv (\neg p \vee \neg q \vee \neg r) \wedge (\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge (p \vee \neg q \vee \neg r)\end{aligned}$$

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Procedure

① eliminate implications

② push negations towards atoms and remove double negations

③ distribute disjunction over conjunction

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Deterministic Procedure

① eliminate implications

function $\text{IMPL\_FREE}(\varphi)$:
begin function
case φ is atom: **return** φ
φ is $\neg\varphi_1$: **return** $\neg\,\text{IMPL\_FREE}(\varphi_1)$
φ is $\varphi_1 \wedge \varphi_2$: **return** $\text{IMPL\_FREE}(\varphi_1) \wedge \text{IMPL\_FREE}(\varphi_2)$
φ is $\varphi_1 \vee \varphi_2$: **return** $\text{IMPL\_FREE}(\varphi_1) \vee \text{IMPL\_FREE}(\varphi_2)$
φ is $\varphi_1 \to \varphi_2$: **return** $\neg\,\text{IMPL\_FREE}(\varphi_1) \vee \text{IMPL\_FREE}(\varphi_2)$
end case
end function

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Deterministic Procedure

② push negations towards atoms and remove double negations

function NNF(φ):

begin function

case	φ is literal:	**return**	φ
	φ is $\neg\neg\varphi_1$:	**return**	$\text{NNF}(\varphi_1)$
	φ is $\varphi_1 \wedge \varphi_2$:	**return**	$\text{NNF}(\varphi_1) \wedge \text{NNF}(\varphi_2)$
	φ is $\varphi_1 \vee \varphi_2$:	**return**	$\text{NNF}(\varphi_1) \vee \text{NNF}(\varphi_2)$
	φ is $\neg(\varphi_1 \wedge \varphi_2)$:	**return**	$\text{NNF}(\neg\varphi_1) \vee \text{NNF}(\neg\varphi_2)$
	φ is $\neg(\varphi_1 \vee \varphi_2)$:	**return**	$\text{NNF}(\neg\varphi_1) \wedge \text{NNF}(\neg\varphi_2)$

end case

end function

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Deterministic Procedure

② push negations towards atoms and remove double negations

function NNF(φ):

begin function

case	φ is literal:	**return**	φ
	φ is $\neg\neg\varphi_1$:	**return**	NNF(φ_1)
	φ is $\varphi_1 \wedge \varphi_2$:	**return**	NNF(φ_1) $\wedge$ NNF(φ_2)
	φ is $\varphi_1 \vee \varphi_2$:	**return**	NNF(φ_1) $\vee$ NNF(φ_2)
	φ is $\neg(\varphi_1 \wedge \varphi_2)$:	**return**	NNF($\neg\varphi_1$) $\vee$ NNF($\neg\varphi_2$)
	φ is $\neg(\varphi_1 \vee \varphi_2)$:	**return**	NNF($\neg\varphi_1$) $\wedge$ NNF($\neg\varphi_2$)

end case

end function

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Deterministic Procedure

③ distribute disjunction over conjunction

function $\mathsf{CNF}(\varphi)$:

begin function

case	φ is literal:	**return**	φ
	φ is $\varphi_1 \wedge \varphi_2$:	**return**	$\mathsf{CNF}(\varphi_1) \wedge \mathsf{CNF}(\varphi_2)$
	φ is $\varphi_1 \vee \varphi_2$:	**return**	$\mathsf{DISTR}(\mathsf{CNF}(\varphi_1), \mathsf{CNF}(\varphi_2))$

end case

end function

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Deterministic Procedure

③ distribute disjunction over conjunction

function CNF(φ):
begin function

case	φ is literal:	**return**	φ
	φ is $\varphi_1 \wedge \varphi_2$:	**return**	$\text{CNF}(\varphi_1) \wedge \text{CNF}(\varphi_2)$
	φ is $\varphi_1 \vee \varphi_2$:	**return**	$\text{DISTR}(\text{CNF}(\varphi_1), \text{CNF}(\varphi_2))$

end case
end function

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Deterministic Procedure

③ distribute disjunction over conjunction

function DISTR(η_1, η_2):

begin function

case	η_1 is $\eta_{11} \wedge \eta_{12}$:	**return**	DISTR$(\eta_{11}, \eta_2) \wedge$ DISTR(η_{12}, η_2)
	η_2 is $\eta_{21} \wedge \eta_{22}$:	**return**	DISTR$(\eta_1, \eta_{21}) \wedge$ DISTR(η_1, η_{22})
	otherwise:	**return**	$\eta_1 \vee \eta_2$

end case

end function

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Deterministic Procedure

① eliminate implications

② push negations towards atoms and remove double negations

③ distribute disjunction over conjunction

Theorem

❶ $\text{CNF}(\text{NNF}(\text{IMPL\_FREE}(\varphi)))$ is CNF

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Deterministic Procedure

① eliminate implications

② push negations towards atoms and remove double negations

③ distribute disjunction over conjunction

Theorem

❶ CNF(NNF(IMPL_FREE(φ))) is CNF

❷ CNF(NNF(IMPL_FREE(φ))) $\equiv \varphi$

Theorem

for every formula φ there exists CNF ψ such that $\varphi \equiv \psi$

Deterministic Procedure

① eliminate implications

② push negations towards atoms and remove double negations

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Theorem

1. $\mathsf{CNF}(\mathsf{NNF}(\mathsf{IMPL\_FREE}(\varphi)))$ is CNF
2. $\mathsf{CNF}(\mathsf{NNF}(\mathsf{IMPL\_FREE}(\varphi))) \equiv \varphi$
3. executing $\mathsf{CNF}(\mathsf{NNF}(\mathsf{IMPL\_FREE}(\varphi)))$ terminates

Outline

Huth and Ryan

- Section 1.1
- Section 1.3
- Sections 1.4.1 and 1.4.2
- Sections 1.5.1 and 1.5.2

Huth and Ryan

- Section 1.1
- Section 1.3
- Sections 1.4.1 and 1.4.2
- Sections 1.5.1 and 1.5.2

Differences (slides – book)

- role of $\bot$ and $\top$
- terminology concerning CNFs

Important Concepts

- atom
- bottom
- clause
- complementary literals
- conjunction
- conjunctive normal form
- disjunction
- disjunctive normal form
- implication
- literal
- negation
- top
- right-associativity
- satisfiability
- semantic entailment
- semantic equivalence
- tautology
- truth table
- truth values
- validity
- valuation

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homework for March 7 (this week)