



Logic

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Outline

- 1. Summary of Previous Lecture**
- 2. Sorting Networks**
- 3. Intermezzo**
- 4. Conflict Graph**
- 5. Further Reading**
- 6. Course Recommendations**
- 7. Exam**

Definition

CTL* formulas consist of **state formulas**, which are evaluated in states:

$$\varphi ::= \perp \mid \top \mid p \mid (\neg\varphi) \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid (\varphi \rightarrow \varphi) \mid \mathbf{A}[\alpha] \mid \mathbf{E}[\alpha]$$

and **path formulas**, which are evaluated along paths:

$$\alpha ::= \varphi \mid (\neg\alpha) \mid (\alpha \wedge \alpha) \mid (\alpha \vee \alpha) \mid (\alpha \rightarrow \alpha) \mid (\mathbf{X}\alpha) \mid (\mathbf{F}\alpha) \mid (\mathbf{G}\alpha) \mid (\alpha \mathbf{U}\alpha)$$

Definition

satisfaction of CTL* **state formula** φ in state $s \in S$ and **path formula** α with respect to path π in model $\mathcal{M} = (S, \rightarrow, L)$ is defined by induction on φ

Theorem

satisfaction of CTL* formulas in finite models is **decidable**

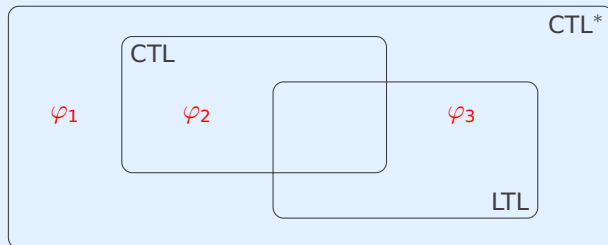
Definition

CTL* state (CTL, LTL) formulas φ and ψ are **semantically equivalent** if

$$\mathcal{M}, s \models \varphi \iff \mathcal{M}, s \models \psi$$

for all models $\mathcal{M} = (S, \rightarrow, L)$ and states $s \in S$

Expressive Power



$$\varphi_1 = E[GF p]$$

$$\varphi_2 = AGEF p$$

$$\varphi_3 = A[GF p \rightarrow F q]$$

Definition (Abstract DPLL)

- ▶ states $M \parallel F$ consist of list M of (annotated) non-complementary literals and CNF F
- ▶ transition rules

- ▶ **unit propagate**
$$M \parallel F, C \vee \ell \implies M \ell \parallel F, C \vee \ell$$

if $M \models \neg C$ and ℓ is undefined in M

- ▶ **pure literal**
$$M \parallel F \implies M \ell \parallel F$$

if ℓ occurs in F and ℓ^c does not occur in F and ℓ is undefined in M

- ▶ **decide**
$$M \parallel F \implies M \overset{d}{\ell} \parallel F$$

if ℓ or ℓ^c occurs in F and ℓ is undefined in M

- ▶ **fail**
$$M \parallel F, C \implies \text{fail-state}$$

if $M \models \neg C$ and M contains no decision literals

Definition (Abstract DPLL, cont'd)

▶ **backtrack** $M \stackrel{d}{\ell} N \parallel F, C \implies M \ell^c \parallel F, C$

if $M \stackrel{d}{\ell} N \models \neg C$ and N contains no decision literals

▶ **backjump** $M \stackrel{d}{\ell} N \parallel F, C \implies M \ell' \parallel F, C$

if $M \stackrel{d}{\ell} N \models \neg C$ and there exists clause $C' \vee \ell'$ such that

▶ $F, C \models C' \vee \ell'$ **backjump clause**

▶ $M \models \neg C'$

▶ ℓ' is undefined in M

▶ ℓ' or ℓ'^c occurs in F or in $M \stackrel{d}{\ell} N$

Definition

basic DPLL \mathcal{B} consists of transition rules **unit propagate**, **decide**, **fail**, **backjump**

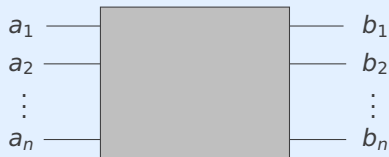
Lemma

if $\| F \Longrightarrow_{\mathcal{B}}^* M_0 \overset{d}{l_1} M_1 \overset{d}{l_2} M_2 \cdots \overset{d}{l_k} M_k \| F$ with no decision literals in M_0, \dots, M_k
then $F, l_1, \dots, l_i \models M_i$ for all $0 \leq i \leq k$

Theorem

- ▶ there are no infinite derivations $\| F \Longrightarrow_{\mathcal{B}} S_1 \Longrightarrow_{\mathcal{B}} S_2 \Longrightarrow_{\mathcal{B}} \cdots$
- ▶ if $\| F \Longrightarrow_{\mathcal{B}} S_1 \Longrightarrow_{\mathcal{B}} \cdots \Longrightarrow_{\mathcal{B}} S_n \not\Longrightarrow_{\mathcal{B}}$ then
 - 1 $S_n = \text{fail-state}$ if and only if F is unsatisfiable
 - 2 $S_n = M \| F'$ only if F is satisfiable and $M \models F$

Comparator Network



Definition

sorting network is comparator network that transforms any input sequence $a = (a_1, \dots, a_n)$ of natural numbers into **sorted** output sequence $b = (b_1, \dots, b_n)$:

b is permutation of a and $b_1 \leq \dots \leq b_n$

Part I: Propositional Logic

algebraic normal forms, binary decision diagrams, conjunctive normal forms, DPLL, Horn formulas, natural deduction, Post's adequacy theorem, resolution, SAT, semantics, sorting networks, soundness and completeness, syntax, Tseitin's transformation

Part II: Predicate Logic

natural deduction, quantifier equivalences, resolution, semantics, Skolemization, syntax, undecidability, unification

Part III: Model Checking

adequacy, branching-time temporal logic, CTL*, fairness, linear-time temporal logic, model checking algorithms, symbolic model checking

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Definition

function f is **monotonic** if $f(x) \leq f(y)$ whenever $x \leq y$

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Lemma

if comparator network N transforms (a_1, \dots, a_n) into (b_1, \dots, b_n) and f is monotonic then N transforms $(f(a_1), \dots, f(a_n))$ into $(f(b_1), \dots, f(b_n))$

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- ▶ induction on size

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sketch

- ▶ induction on size
- ▶ if $x \leq y$ then $f(x) \leq f(y)$ and thus

$$\min(f(x), f(y)) = f(\min(x, y))$$

$$\max(f(x), f(y)) = f(\max(x, y))$$

Theorem (Zero-One Principle)

it suffices to test all 2^n sequences consisting of n zeros and ones

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$$f(x) = \begin{cases} 0 & \text{if } x \leq a_i \\ 1 & \text{if } x > a_i \end{cases}$$

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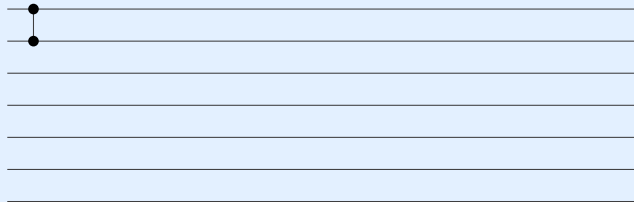
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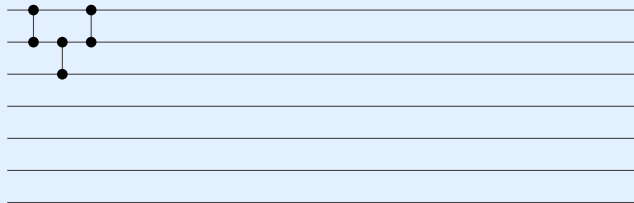
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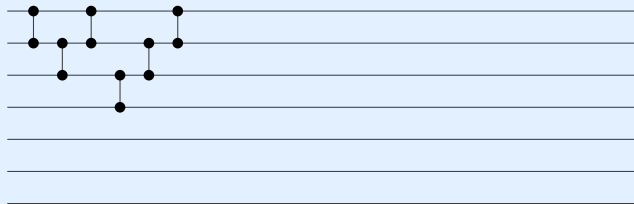
Insertion Sort



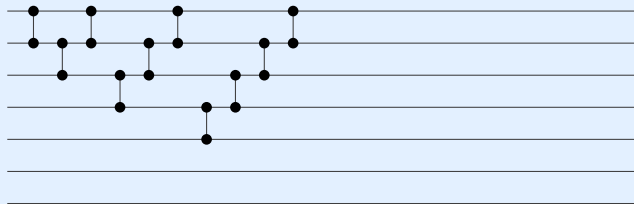
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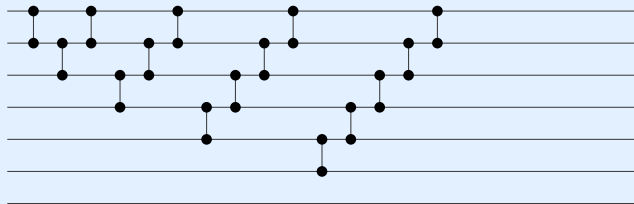
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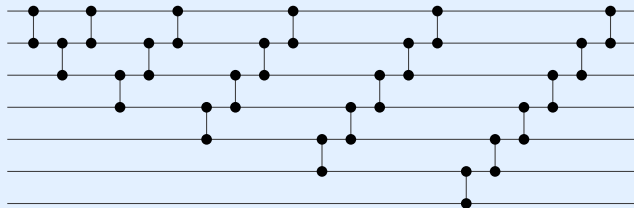
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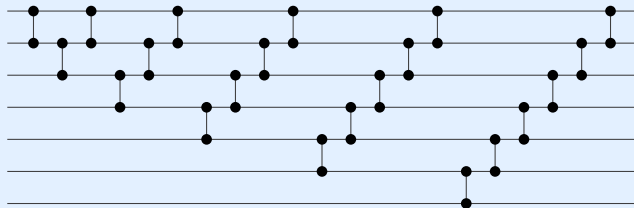
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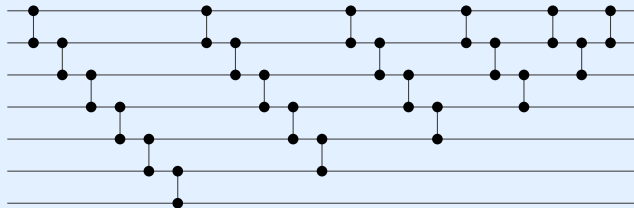
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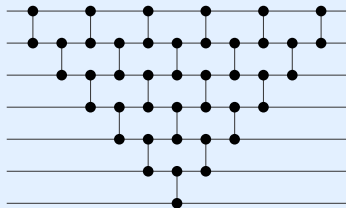
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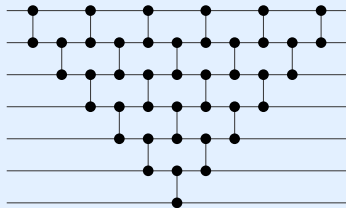
Bubble Sort



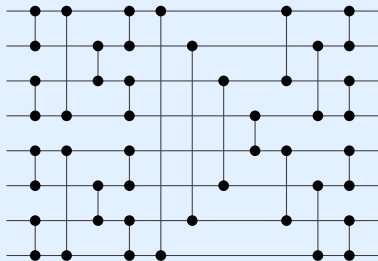
Insertion Sort



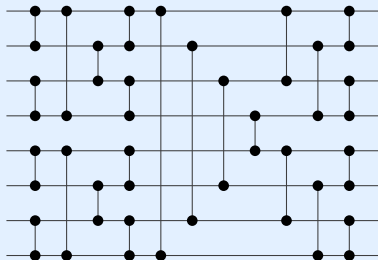
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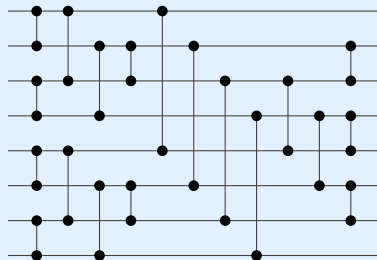
Bitonic Sort



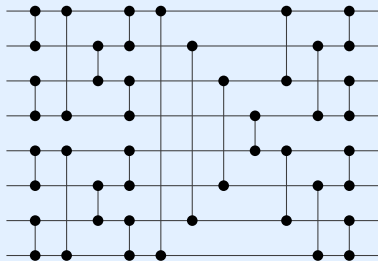
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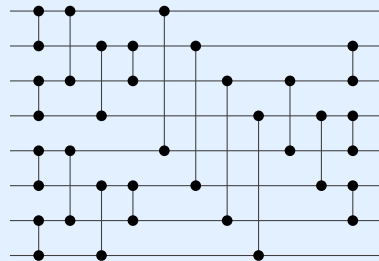
Odd-Even Mergesort



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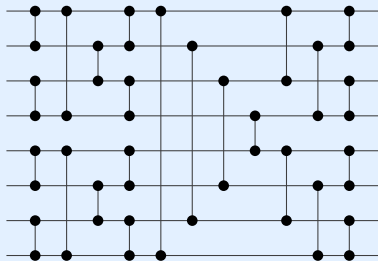
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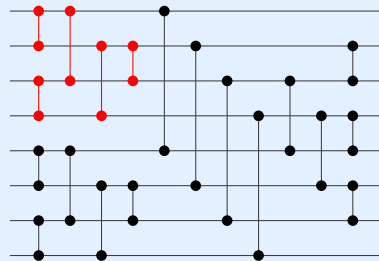
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to sort (a_1, \dots, a_n) with $n = 2^m$ for some $m > 1$

Bitonic Sort



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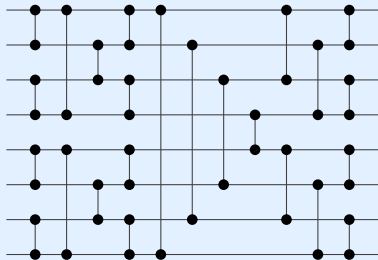


Odd-Even Mergesort

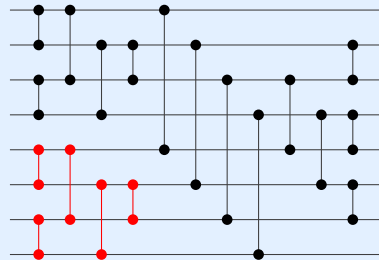
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- ① recursively sort $(a_1, \dots, a_{n/2})$

Bitonic Sort



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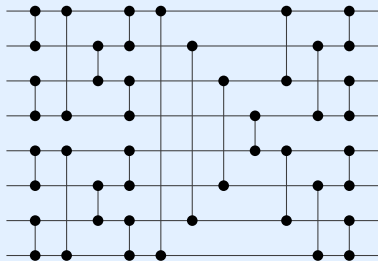


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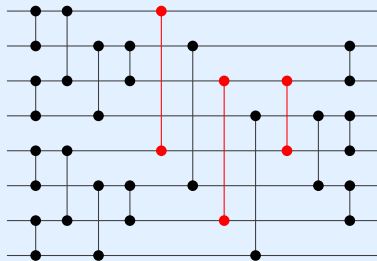
to sort (a_1, \dots, a_n) with $n = 2^m$ for some $m > 1$

① recursively sort $(a_1, \dots, a_{n/2})$ and $(a_{n/2+1}, \dots, a_n)$

Bitonic Sort



Odd-Even Mergesort

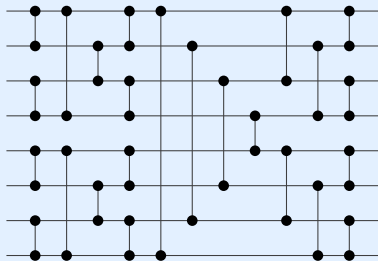


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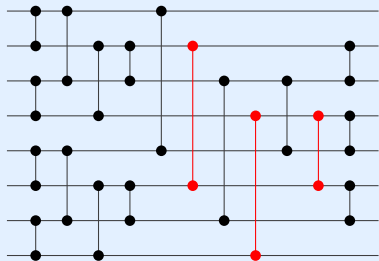
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- ① recursively sort $(a_1, \dots, a_{n/2})$ and $(a_{n/2+1}, \dots, a_n)$
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Bitonic Sort



Odd-Even Mergesort

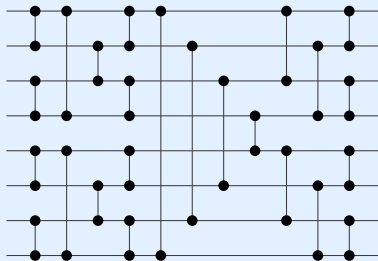


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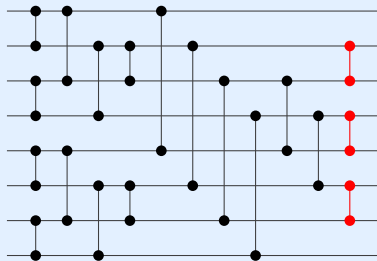
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Bitonic Sort



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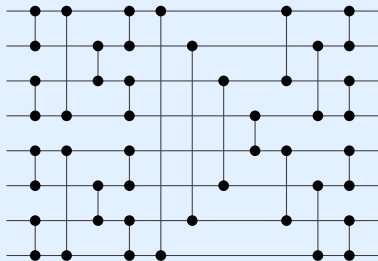


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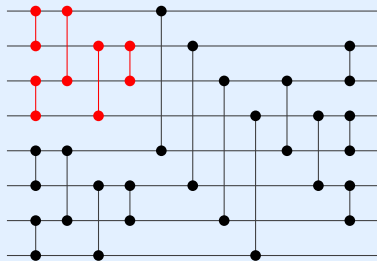
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- ③ compare $(a_2, a_3), (a_4, a_5), \dots, (a_{n-2}, a_{n-1})$

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Complexity

sorting n inputs requires

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- | | | |
|-----------------------|-------------------------------|--------------------------------|
| ▶ insertion sort: | depth $2n - 3$ | size $\frac{n(n-1)}{2}$ |
| ▶ bitonic sort: | depth $\mathcal{O}(\log^2 n)$ | size $\mathcal{O}(n \log^2 n)$ |
| ▶ odd-even mergesort: | depth $\mathcal{O}(\log^2 n)$ | size $\mathcal{O}(n \log^2 n)$ |

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Known Bounds on Depth

wires 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17

Known Bounds on Depth

# wires	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
upper bound	0	1	3	3	5	5	6	6	7	7	8	8	9	9	9	9	10

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Known Bounds on Size

# wires	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
upper bound	0	1	3	5	9	12	16	19	25	29	35	39	45	51	56	60	71

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Known Bounds on Depth

# wires	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
upper bound	0	1	3	3	5	5	6	6	7	7	8	8	9	9	9	9	10
lower bound	0	1	3	3	5	5	6	6	7	7	8	8	9	9	9	9	10

Known Bounds on Size

# wires	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
upper bound	0	1	3	5	9	12	16	19	25	29	35	39	45	51	56	60	71
lower bound	0	1	3	5	9	12	16	19	25	29	33	37	41	45	49	53	58

Known Bounds on Depth

# wires	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
upper bound	0	1	3	3	5	5	6	6	7	7	8	8	9	9	9	9	10
lower bound	0	1	3	3	5	5	6	6	7	7	8	8	9	9	9	9	10

Known Bounds on Size

# wires	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
upper bound	0	1	3	5	9	12	16	19	25	29	35	39	45	51	56	60	71
lower bound	0	1	3	5	9	12	16	19	25	29	33	37	41	45	49	53	58

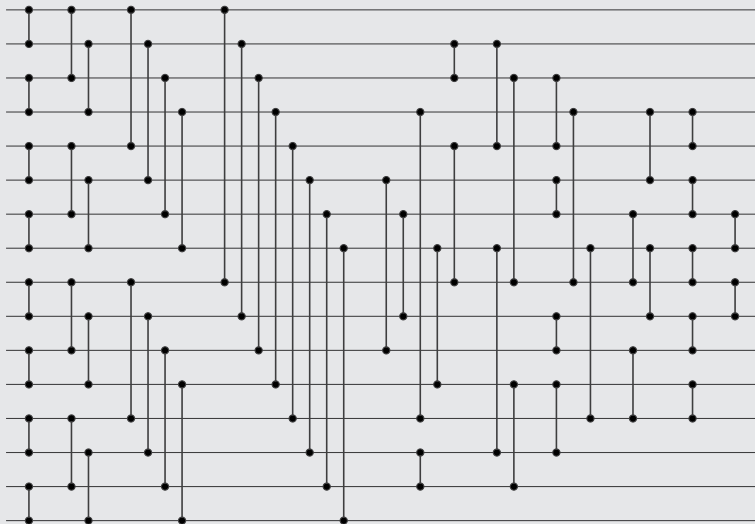
Known Bounds on Depth

# wires	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
upper bound	0	1	3	3	5	5	6	6	7	7	8	8	9	9	9	9	10
lower bound	0	1	3	3	5	5	6	6	7	7	8	8	9	9	9	9	10

Known Bounds on Size

# wires	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
upper bound	0	1	3	5	9	12	16	19	25	29	35	39	45	51	56	60	71
lower bound	0	1	3	5	9	12	16	19	25	29	33	37	41	45	49	53	58

Example (16 wires, size 60)



Outline

1. Summary of Previous Lecture

2. Sorting Networks

Zero-One Principle

Construction

Optimality

Verification

3. Intermezzo

4. Conflict Graph

5. Further Reading

6. Course Recommendations

7. Exam

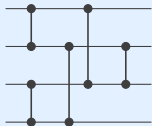
Question

how to verify that comparator network is sorting network ?

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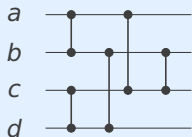
SAT Encoding



Question

how to verify that comparator network is sorting network ?

SAT Encoding

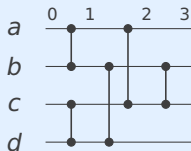


variables x_i with $x \in \{a, b, c, d\}$

Question

how to verify that comparator network is sorting network ?

SAT Encoding

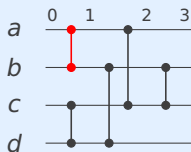


variables x_i with $x \in \{a, b, c, d\}$ and $i \in \{0, 1, 2, 3\}$

Question

how to verify that comparator network is sorting network ?

SAT Encoding



variables x_i with $x \in \{a, b, c, d\}$ and $i \in \{0, 1, 2, 3\}$

constraints C

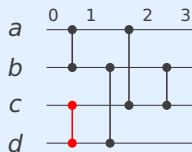
$$a_1 \leftrightarrow a_0 \wedge b_0$$

$$b_1 \leftrightarrow a_0 \vee b_0$$

Question

how to verify that comparator network is sorting network ?

SAT Encoding



variables x_i with $x \in \{a, b, c, d\}$ and $i \in \{0, 1, 2, 3\}$

constraints C

$$a_1 \leftrightarrow a_0 \wedge b_0$$

$$b_1 \leftrightarrow a_0 \vee b_0$$

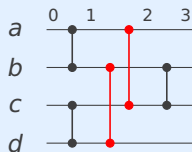
$$c_1 \leftrightarrow c_0 \wedge d_0$$

$$d_1 \leftrightarrow c_0 \vee d_0$$

Question

how to verify that comparator network is sorting network ?

SAT Encoding



variables x_i with $x \in \{a, b, c, d\}$ and $i \in \{0, 1, 2, 3\}$

constraints C

$$a_1 \leftrightarrow a_0 \wedge b_0$$

$$a_2 \leftrightarrow a_1 \wedge c_1$$

$$b_1 \leftrightarrow a_0 \vee b_0$$

$$b_2 \leftrightarrow b_1 \wedge d_1$$

$$c_1 \leftrightarrow c_0 \wedge d_0$$

$$c_2 \leftrightarrow a_1 \vee c_1$$

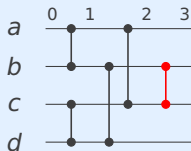
$$d_1 \leftrightarrow c_0 \vee d_0$$

$$d_2 \leftrightarrow b_1 \vee d_1$$

Question

how to verify that comparator network is sorting network ?

SAT Encoding



variables x_i with $x \in \{a, b, c, d\}$ and $i \in \{0, 1, 2, 3\}$

constraints C

$$a_1 \leftrightarrow a_0 \wedge b_0$$

$$b_1 \leftrightarrow a_0 \vee b_0$$

$$c_1 \leftrightarrow c_0 \wedge d_0$$

$$d_1 \leftrightarrow c_0 \vee d_0$$

$$a_2 \leftrightarrow a_1 \wedge c_1$$

$$b_2 \leftrightarrow b_1 \wedge d_1$$

$$c_2 \leftrightarrow a_1 \vee c_1$$

$$d_2 \leftrightarrow b_1 \vee d_1$$

$$a_3 \leftrightarrow a_2$$

$$b_3 \leftrightarrow b_2 \wedge c_2$$

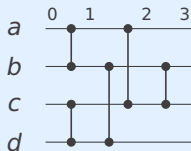
$$c_3 \leftrightarrow b_2 \vee c_2$$

$$d_3 \leftrightarrow d_2$$

Question

how to verify that comparator network is sorting network ?

SAT Encoding



variables x_i with $x \in \{a, b, c, d\}$ and $i \in \{0, 1, 2, 3\}$

constraints C

$$a_1 \leftrightarrow a_0 \wedge b_0$$

$$a_2 \leftrightarrow a_1 \wedge c_1$$

$$a_3 \leftrightarrow a_2$$

$$b_1 \leftrightarrow a_0 \vee b_0$$

$$b_2 \leftrightarrow b_1 \wedge d_1$$

$$b_3 \leftrightarrow b_2 \wedge c_2$$

$$c_1 \leftrightarrow c_0 \wedge d_0$$

$$c_2 \leftrightarrow a_1 \vee c_1$$

$$c_3 \leftrightarrow b_2 \vee c_2$$

$$d_1 \leftrightarrow c_0 \vee d_0$$

$$d_2 \leftrightarrow b_1 \vee d_1$$

$$d_3 \leftrightarrow d_2$$

sorting network $\iff C \wedge ((a_3 \wedge \neg b_3) \vee (b_3 \wedge \neg c_3) \vee (c_3 \wedge \neg d_3))$ is unsatisfiable

Demo

SNV (sorting network visualizer)

by Rick Spiegl (2016)

Demo

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by Rick Spiegl (2016)

Remark

sorting networks are used in **pseudo-boolean constraint solving**

Demo

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Remark

sorting networks are used in **pseudo-boolean constraint solving**

$$x_1 + \dots + x_n \geq 1$$

Demo

SNV (sorting network visualizer)

by Rick Spiegl (2016)

Remark

sorting networks are used in **pseudo-boolean constraint solving**

$$x_1 + \cdots + x_n \geq 1$$

$$x_1 + \cdots + x_n \leq 1$$

Outline

1. Summary of Previous Lecture
2. Sorting Networks
- 3. Intermezzo**
4. Conflict Graph
5. Further Reading
6. Course Recommendations
7. Exam

Question

Which of the following statements are true ?

- A** The function $f(x) = x^2$ is monotonic on the integers.
- B** Given 2^n wires, there is a comparator network of depth n which computes minimum and maximum.
- C** There is a sorting network for 20 wires with size 190 and depth 37.
- D** The depth-optimal sorting network for a given size is unique.
- E** If the sorting network constraint is satisfiable, an input which does not get sorted can be derived from a satisfying assignment.



Outline

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Example

ア	1
イ	$2 \vee \neg 3$
ウ	$2 \vee \neg 4$
エ	$3 \vee \neg 12 \vee \neg 13$
オ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$
カ	$\neg 5 \vee \neg 6$
キ	$6 \vee 7$
ク	$6 \vee \neg 12 \vee 14$
ケ	$\neg 7 \vee 16 \vee 17$
コ	$\neg 8 \vee 9$
サ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$
シ	$\neg 9 \vee \neg 19 \vee \neg 21$
ス	$10 \vee 11$
セ	$13 \vee \neg 14 \vee \neg 15$
ソ	$16 \vee \neg 18$
タ	$16 \vee 19$
チ	$\neg 19 \vee 20$

Example

α	1	$\frac{1}{\alpha}$
β	$2 \vee \neg 3$	
γ	$2 \vee \neg 4$	
δ	$3 \vee \neg 12 \vee \neg 13$	
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$	
ζ	$\neg 5 \vee \neg 6$	
η	$6 \vee 7$	
θ	$6 \vee \neg 12 \vee 14$	
ι	$\neg 7 \vee 16 \vee 17$	
κ	$\neg 8 \vee 9$	
λ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$	
μ	$\neg 9 \vee \neg 19 \vee \neg 21$	
ν	$10 \vee 11$	
ξ	$13 \vee \neg 14 \vee \neg 15$	
\omicron	$16 \vee \neg 18$	
π	$16 \vee 19$	
ρ	$\neg 19 \vee 20$	

Example

α	1	$1 \overset{d}{\neg} 2$
β	$2 \vee \neg 3$	
γ	$2 \vee \neg 4$	
δ	$3 \vee \neg 12 \vee \neg 13$	
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$	
ζ	$\neg 5 \vee \neg 6$	
η	$6 \vee 7$	
θ	$6 \vee \neg 12 \vee 14$	
ι	$\neg 7 \vee 16 \vee 17$	
κ	$\neg 8 \vee 9$	
λ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$	
μ	$\neg 9 \vee \neg 19 \vee \neg 21$	
ν	$10 \vee 11$	
ξ	$13 \vee \neg 14 \vee \neg 15$	
\omicron	$16 \vee \neg 18$	
π	$16 \vee 19$	
ρ	$\neg 19 \vee 20$	

Example

α	1	$1 \overset{d}{\neg 2} \neg 3$
β	$2 \vee \neg 3$	$\alpha \quad \beta$
γ	$2 \vee \neg 4$	
δ	$3 \vee \neg 12 \vee \neg 13$	
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$	
ζ	$\neg 5 \vee \neg 6$	
η	$6 \vee 7$	
θ	$6 \vee \neg 12 \vee 14$	
ι	$\neg 7 \vee 16 \vee 17$	
κ	$\neg 8 \vee 9$	
λ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$	
μ	$\neg 9 \vee \neg 19 \vee \neg 21$	
ν	$10 \vee 11$	
ξ	$13 \vee \neg 14 \vee \neg 15$	
\omicron	$16 \vee \neg 18$	
π	$16 \vee 19$	
ρ	$\neg 19 \vee 20$	

Example

α	1	1	$\overset{d}{-2}$	-3	-4
β	$2 \vee -3$	α		β	γ
γ	$2 \vee -4$				
δ	$3 \vee -12 \vee -13$				
ϵ	$4 \vee -17 \vee 18 \vee -19 \vee 21$				
ζ	$-5 \vee -6$				
η	$6 \vee 7$				
θ	$6 \vee -12 \vee 14$				
ι	$-7 \vee 16 \vee 17$				
κ	$-8 \vee 9$				
λ	$-8 \vee -11 \vee 15 \vee -16$				
μ	$-9 \vee -19 \vee -21$				
ν	$10 \vee 11$				
ξ	$13 \vee -14 \vee -15$				
\omicron	$16 \vee -18$				
π	$16 \vee 19$				
ρ	$-19 \vee 20$				

Example

α	1	1	$\overset{d}{-2}$	$\overset{d}{-3}$	$\overset{d}{-4}$	$\overset{d}{5}$
β	$2 \vee -3$	α		β	γ	
γ	$2 \vee -4$					
δ	$3 \vee -12 \vee -13$					
ϵ	$4 \vee -17 \vee 18 \vee -19 \vee 21$					
ζ	$-5 \vee -6$					
η	$6 \vee 7$					
θ	$6 \vee -12 \vee 14$					
ι	$-7 \vee 16 \vee 17$					
κ	$-8 \vee 9$					
λ	$-8 \vee -11 \vee 15 \vee -16$					
μ	$-9 \vee -19 \vee -21$					
ν	$10 \vee 11$					
ξ	$13 \vee -14 \vee -15$					
\omicron	$16 \vee -18$					
π	$16 \vee 19$					
ρ	$-19 \vee 20$					

Example

α	1	1	$\overset{d}{-2}$	$\overset{d}{-3}$	$\overset{d}{-4}$	$\overset{d}{5}$	$\overset{d}{-6}$
β	$2 \vee -3$	α		β	γ		ζ
γ	$2 \vee -4$						
δ	$3 \vee -12 \vee -13$						
ϵ	$4 \vee -17 \vee 18 \vee -19 \vee 21$						
ζ	$-5 \vee -6$						
η	$6 \vee 7$						
θ	$6 \vee -12 \vee 14$						
ι	$-7 \vee 16 \vee 17$						
κ	$-8 \vee 9$						
λ	$-8 \vee -11 \vee 15 \vee -16$						
μ	$-9 \vee -19 \vee -21$						
ν	$10 \vee 11$						
ξ	$13 \vee -14 \vee -15$						
\omicron	$16 \vee -18$						
π	$16 \vee 19$						
ρ	$-19 \vee 20$						

Example

α	1				1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7
β	$2 \vee \neg 3$				α		β	γ		ζ	η
γ	$2 \vee \neg 4$										
δ	$3 \vee \neg 12 \vee \neg 13$										
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$										
ζ	$\neg 5 \vee \neg 6$										
η	$6 \vee 7$										
θ	$6 \vee \neg 12 \vee 14$										
ι	$\neg 7 \vee 16 \vee 17$										
κ	$\neg 8 \vee 9$										
λ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$										
μ	$\neg 9 \vee \neg 19 \vee \neg 21$										
ν	$10 \vee 11$										
ξ	$13 \vee \neg 14 \vee \neg 15$										
\omicron	$16 \vee \neg 18$										
π	$16 \vee 19$										
ρ	$\neg 19 \vee 20$										

Example

α	1	$1 \overset{d}{\neg} 2 \overset{d}{\neg} 3 \overset{d}{\neg} 4 \overset{d}{5} \overset{d}{\neg} 6 \overset{d}{7} \overset{d}{8}$	decision level 3			
β	$2 \vee \neg 3$	α	β	γ	ζ	η
γ	$2 \vee \neg 4$					
δ	$3 \vee \neg 12 \vee \neg 13$					
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$					
ζ	$\neg 5 \vee \neg 6$					
η	$6 \vee 7$					
θ	$6 \vee \neg 12 \vee 14$					
ι	$\neg 7 \vee 16 \vee 17$					
κ	$\neg 8 \vee 9$					
λ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$					
μ	$\neg 9 \vee \neg 19 \vee \neg 21$					
ν	$10 \vee 11$					
ξ	$13 \vee \neg 14 \vee \neg 15$					
\omicron	$16 \vee \neg 18$					
π	$16 \vee 19$					
ρ	$\neg 19 \vee 20$					

Example

α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9
β	$2 \vee \neg 3$	α		β	γ		ζ	η	κ	
γ	$2 \vee \neg 4$									
δ	$3 \vee \neg 12 \vee \neg 13$									
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$									
ζ	$\neg 5 \vee \neg 6$									
η	$6 \vee 7$									
θ	$6 \vee \neg 12 \vee 14$									
ι	$\neg 7 \vee 16 \vee 17$									
κ	$\neg 8 \vee 9$									
λ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$									
μ	$\neg 9 \vee \neg 19 \vee \neg 21$									
ν	$10 \vee 11$									
ξ	$13 \vee \neg 14 \vee \neg 15$									
\omicron	$16 \vee \neg 18$									
π	$16 \vee 19$									
ρ	$\neg 19 \vee 20$									

Example

α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	decision level 4
β	$2 \vee \neg 3$	α		β	γ		ζ	η	κ			
γ	$2 \vee \neg 4$											
δ	$3 \vee \neg 12 \vee \neg 13$											
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$											
ζ	$\neg 5 \vee \neg 6$											
η	$6 \vee 7$											
θ	$6 \vee \neg 12 \vee 14$											
ι	$\neg 7 \vee 16 \vee 17$											
κ	$\neg 8 \vee 9$											
λ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$											
μ	$\neg 9 \vee \neg 19 \vee \neg 21$											
ν	$10 \vee 11$											
ξ	$13 \vee \neg 14 \vee \neg 15$											
\omicron	$16 \vee \neg 18$											
π	$16 \vee 19$											
ρ	$\neg 19 \vee 20$											

Example

α	1				$1 \overset{d}{\neg 2}$	$\neg 3$	$\neg 4$	$5 \overset{d}{\neg 6}$	$7 \overset{d}{8}$	$9 \overset{d}{\neg 10}$	11
β	$2 \vee \neg 3$				α	β	γ	ζ	η	κ	ν
γ	$2 \vee \neg 4$										
δ	$3 \vee \neg 12 \vee \neg 13$										
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$										
ζ	$\neg 5 \vee \neg 6$										
η	$6 \vee 7$										
θ	$6 \vee \neg 12 \vee 14$										
ι	$\neg 7 \vee 16 \vee 17$										
κ	$\neg 8 \vee 9$										
λ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$										
μ	$\neg 9 \vee \neg 19 \vee \neg 21$										
ν	$10 \vee 11$										
ξ	$13 \vee \neg 14 \vee \neg 15$										
\omicron	$16 \vee \neg 18$										
π	$16 \vee 19$										
ρ	$\neg 19 \vee 20$										

Example

α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	decision level 5
β	$2 \vee \neg 3$	α		β	γ		ζ	η	κ		ν			
γ	$2 \vee \neg 4$													
δ	$3 \vee \neg 12 \vee \neg 13$													
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$													
ζ	$\neg 5 \vee \neg 6$													
η	$6 \vee 7$													
θ	$6 \vee \neg 12 \vee 14$													
ι	$\neg 7 \vee 16 \vee 17$													
κ	$\neg 8 \vee 9$													
λ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$													
μ	$\neg 9 \vee \neg 19 \vee \neg 21$													
ν	$10 \vee 11$													
ξ	$13 \vee \neg 14 \vee \neg 15$													
\omicron	$16 \vee \neg 18$													
π	$16 \vee 19$													
ρ	$\neg 19 \vee 20$													

Example

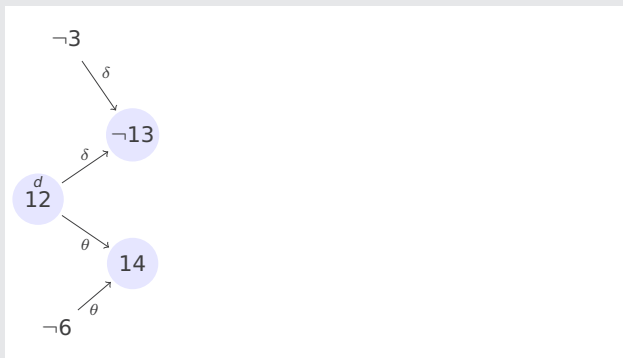
α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	$\neg 13$
β	$2 \vee \neg 3$	α	β	γ	ζ	η	κ	ν	δ					
γ	$2 \vee \neg 4$													
δ	$3 \vee \neg 12 \vee \neg 13$													
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$													
ζ	$\neg 5 \vee \neg 6$													
η	$6 \vee 7$													
θ	$6 \vee \neg 12 \vee 14$													
ι	$\neg 7 \vee 16 \vee 17$													
κ	$\neg 8 \vee 9$													
λ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$													
μ	$\neg 9 \vee \neg 19 \vee \neg 21$													
ν	$10 \vee 11$													
ξ	$13 \vee \neg 14 \vee \neg 15$													
\omicron	$16 \vee \neg 18$													
π	$16 \vee 19$													
ρ	$\neg 19 \vee 20$													



implication graph

Example

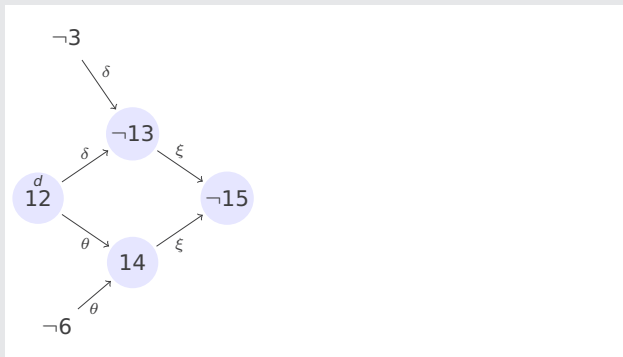
α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	$\neg 13$	14
β	$2 \vee \neg 3$	α		β	γ		ζ	η	κ		ν		δ	θ	
γ	$2 \vee \neg 4$														
δ	$3 \vee \neg 12 \vee \neg 13$														
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$														
ζ	$\neg 5 \vee \neg 6$														
η	$6 \vee 7$														
θ	$6 \vee \neg 12 \vee 14$														
ι	$\neg 7 \vee 16 \vee 17$														
κ	$\neg 8 \vee 9$														
λ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$														
μ	$\neg 9 \vee \neg 19 \vee \neg 21$														
ν	$10 \vee 11$														
ξ	$13 \vee \neg 14 \vee \neg 15$														
\omicron	$16 \vee \neg 18$														
π	$16 \vee 19$														
ρ	$\neg 19 \vee 20$														



implication graph

Example

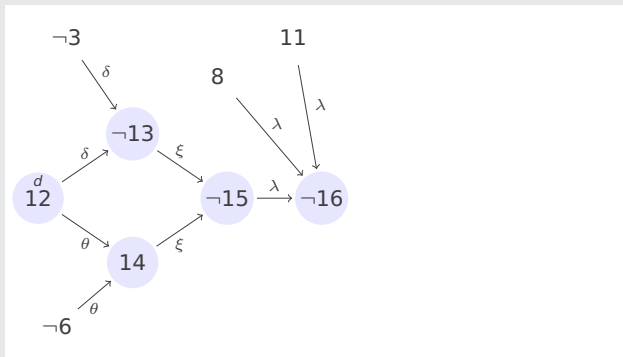
α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	$\neg 13$	14	$\neg 15$
β	$2 \vee \neg 3$	α		β	γ		ζ	η	κ		ν		δ	θ	ξ	
γ	$2 \vee \neg 4$															
δ	$3 \vee \neg 12 \vee \neg 13$															
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$															
ζ	$\neg 5 \vee \neg 6$															
η	$6 \vee 7$															
θ	$6 \vee \neg 12 \vee 14$															
ι	$\neg 7 \vee 16 \vee 17$															
κ	$\neg 8 \vee 9$															
λ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$															
μ	$\neg 9 \vee \neg 19 \vee \neg 21$															
ν	$10 \vee 11$															
ξ	$13 \vee \neg 14 \vee \neg 15$															
\omicron	$16 \vee \neg 18$															
π	$16 \vee 19$															
ρ	$\neg 19 \vee 20$															



implication graph

Example

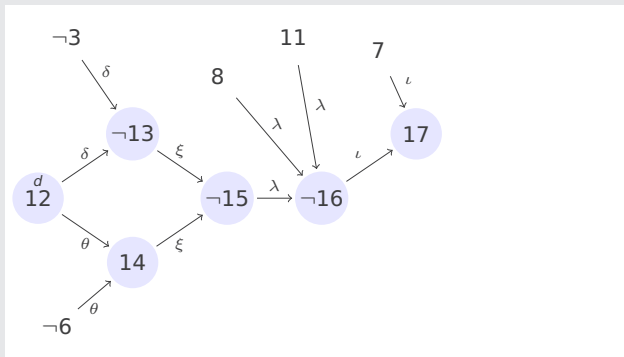
α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	$\neg 13$	14	$\neg 15$	$\neg 16$
β	$2 \vee \neg 3$	α		β	γ		ζ	η	κ		ν		δ	θ	ξ	λ	
γ	$2 \vee \neg 4$																
δ	$3 \vee \neg 12 \vee \neg 13$																
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$																
ζ	$\neg 5 \vee \neg 6$																
η	$6 \vee 7$																
θ	$6 \vee \neg 12 \vee 14$																
ι	$\neg 7 \vee 16 \vee 17$																
κ	$\neg 8 \vee 9$																
λ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$																
μ	$\neg 9 \vee \neg 19 \vee \neg 21$																
ν	$10 \vee 11$																
ξ	$13 \vee \neg 14 \vee \neg 15$																
\omicron	$16 \vee \neg 18$																
π	$16 \vee 19$																
ρ	$\neg 19 \vee 20$																



implication graph

Example

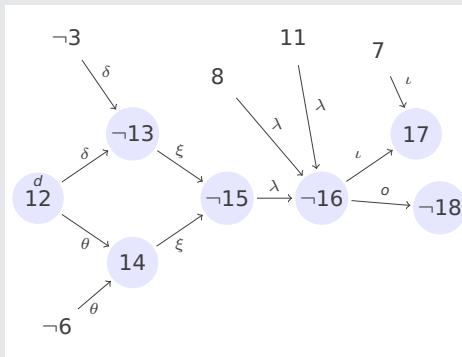
α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	$\neg 13$	14	$\neg 15$	$\neg 16$	17
β		$2 \vee \neg 3$																
γ		$2 \vee \neg 4$																
δ		$3 \vee \neg 12 \vee \neg 13$																
ϵ		$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$																
ζ		$\neg 5 \vee \neg 6$																
η		$6 \vee 7$																
θ		$6 \vee \neg 12 \vee 14$																
ι		$\neg 7 \vee 16 \vee 17$																
κ		$\neg 8 \vee 9$																
λ		$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$																
μ		$\neg 9 \vee \neg 19 \vee \neg 21$																
ν		$10 \vee 11$																
ξ		$13 \vee \neg 14 \vee \neg 15$																
\omicron		$16 \vee \neg 18$																
π		$16 \vee 19$																
ρ		$\neg 19 \vee 20$																



implication graph

Example

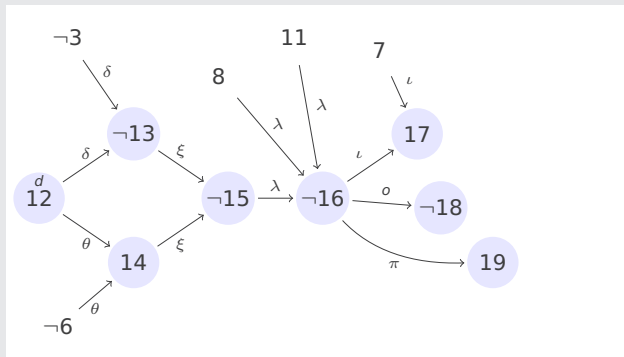
α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	$\neg 13$	14	$\neg 15$	$\neg 16$	17	$\neg 18$
β		$2 \vee$	$\neg 3$																
γ		$2 \vee$	$\neg 4$																
δ		3	$\vee \neg 12$	$\vee \neg 13$															
ϵ		4	$\vee \neg 17$	$\vee 18$	$\vee \neg 19$	$\vee 21$													
ζ		$\neg 5$	$\vee \neg 6$																
η		6	$\vee 7$																
θ		6	$\vee \neg 12$	$\vee 14$															
ι		$\neg 7$	$\vee 16$	$\vee 17$															
κ		$\neg 8$	$\vee 9$																
λ		$\neg 8$	$\vee \neg 11$	$\vee 15$	$\vee \neg 16$														
μ		$\neg 9$	$\vee \neg 19$	$\vee \neg 21$															
ν		10	$\vee 11$																
ξ		13	$\vee \neg 14$	$\vee \neg 15$															
\omicron		16	$\vee \neg 18$																
π		16	$\vee 19$																
ρ		$\neg 19$	$\vee 20$																



implication graph

Example

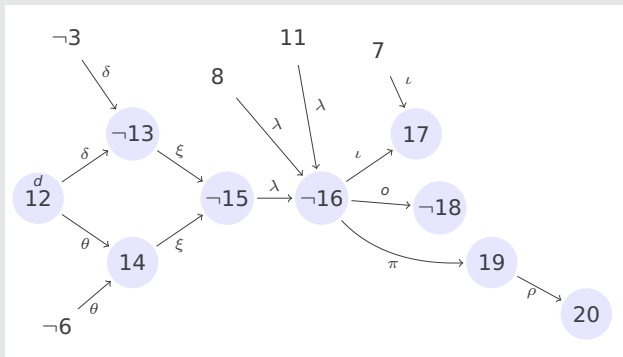
α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	$\neg 13$	14	$\neg 15$	$\neg 16$	17	$\neg 18$	19
β		$2 \vee \neg 3$																		
γ		$2 \vee \neg 4$																		
δ		$3 \vee \neg 12 \vee \neg 13$																		
ϵ		$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$																		
ζ		$\neg 5 \vee \neg 6$																		
η		$6 \vee 7$																		
θ		$6 \vee \neg 12 \vee 14$																		
ι		$\neg 7 \vee 16 \vee 17$																		
κ		$\neg 8 \vee 9$																		
λ		$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$																		
μ		$\neg 9 \vee \neg 19 \vee \neg 21$																		
ν		$10 \vee 11$																		
ξ		$13 \vee \neg 14 \vee \neg 15$																		
\omicron		$16 \vee \neg 18$																		
π		$16 \vee 19$																		
ρ		$\neg 19 \vee 20$																		



implication graph

Example

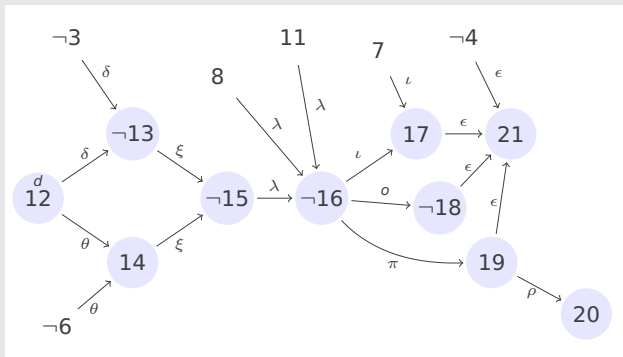
α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	$\neg 9$	10	11	12	$\neg 13$	14	$\neg 15$	$\neg 16$	17	$\neg 18$	19	20
β	$2 \vee \neg 3$	α	β	γ	ζ	η	κ	ν	δ	θ	ξ	λ	ι	\omicron	π	ρ					
γ	$2 \vee \neg 4$																				
δ	$3 \vee \neg 12 \vee \neg 13$																				
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$																				
ζ	$\neg 5 \vee \neg 6$																				
η	$6 \vee 7$																				
θ	$6 \vee \neg 12 \vee 14$																				
ι	$\neg 7 \vee 16 \vee 17$																				
κ	$\neg 8 \vee 9$																				
λ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$																				
μ	$\neg 9 \vee \neg 19 \vee \neg 21$																				
ν	$10 \vee 11$																				
ξ	$13 \vee \neg 14 \vee \neg 15$																				
\omicron	$16 \vee \neg 18$																				
π	$16 \vee 19$																				
ρ	$\neg 19 \vee 20$																				



implication graph

Example

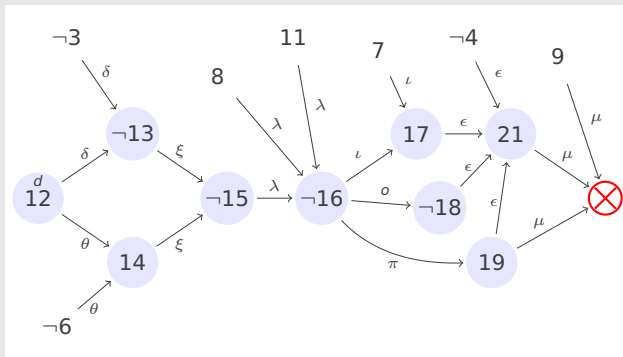
α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	$\neg 13$	14	$\neg 15$	$\neg 16$	17	$\neg 18$	19	20	21
β	$2 \vee \neg 3$	α		β	γ		ζ	η	κ		ν			δ	θ	ξ	λ	ι	\omicron	π	ρ	ϵ
γ	$2 \vee \neg 4$																					
δ	$3 \vee \neg 12 \vee \neg 13$																					
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee \mathbf{21}$																					
ζ	$\neg 5 \vee \neg 6$																					
η	$6 \vee \mathbf{7}$																					
θ	$6 \vee \neg 12 \vee \mathbf{14}$																					
ι	$\neg 7 \vee 16 \vee \mathbf{17}$																					
κ	$\neg 8 \vee \mathbf{9}$																					
λ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$																					
μ	$\neg 9 \vee \neg 19 \vee \neg 21$																					
ν	$10 \vee \mathbf{11}$																					
ξ	$13 \vee \neg 14 \vee \neg 15$																					
\omicron	$16 \vee \neg 18$																					
π	$16 \vee \mathbf{19}$																					
ρ	$\neg 19 \vee \mathbf{20}$																					



implication graph

Example

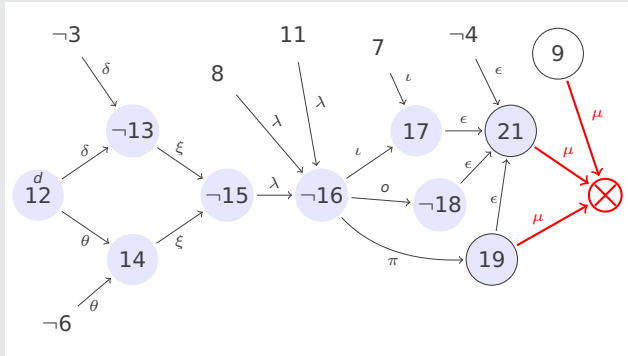
α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	$\neg 13$	14	$\neg 15$	$\neg 16$	17	$\neg 18$	19	20	21
β		$2 \vee$	$\neg 3$																			
γ		$2 \vee$	$\neg 4$																			
δ		3	\vee	$\neg 12$	\vee	$\neg 13$																
ϵ		4	\vee	$\neg 17$	\vee	18	\vee	$\neg 19$	\vee	21												
ζ		$\neg 5$	\vee	$\neg 6$																		
η		6	\vee	7																		
θ		6	\vee	$\neg 12$	\vee	14																
ι		$\neg 7$	\vee	16	\vee	17																
κ		$\neg 8$	\vee	9																		
λ		$\neg 8$	\vee	$\neg 11$	\vee	15	\vee	$\neg 16$														
μ		$\neg 9$	\vee	$\neg 19$	\vee	$\neg 21$																
ν		10	\vee	11																		
ξ		13	\vee	$\neg 14$	\vee	$\neg 15$																
\omicron		16	\vee	$\neg 18$																		
π		16	\vee	19																		
ρ		$\neg 19$	\vee	20																		



conflict graph

Example

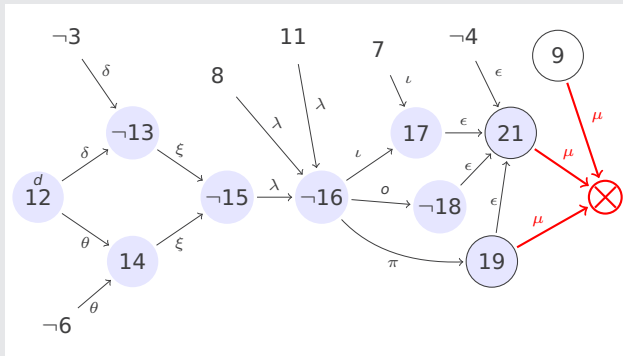
α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	$\neg 13$	14	$\neg 15$	$\neg 16$	17	$\neg 18$	19	20	21
β	$2 \vee \neg 3$	α		β	γ		ζ	η	κ		ν		d	δ	θ	ξ	λ	ι	\omicron	π	ρ	ϵ
γ	$2 \vee \neg 4$																					
δ	$3 \vee \neg 12 \vee \neg 13$																					
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$																					
ζ	$\neg 5 \vee \neg 6$																					
η	$6 \vee 7$																					
θ	$6 \vee \neg 12 \vee 14$																					
ι	$\neg 7 \vee 16 \vee 17$																					
κ	$\neg 8 \vee 9$																					
λ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$																					
μ	$\neg 9 \vee \neg 19 \vee \neg 21$																					
ν	$10 \vee 11$																					
ξ	$13 \vee \neg 14 \vee \neg 15$																					
\omicron	$16 \vee \neg 18$																					
π	$16 \vee 19$																					
ρ	$\neg 19 \vee 20$																					



$\neg 9 \vee \neg 19 \vee \neg 21$ conflict clause (μ)

Example

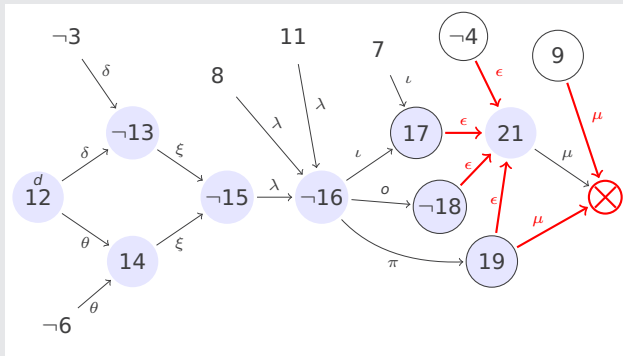
α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	$\neg 13$	14	$\neg 15$	$\neg 16$	17	$\neg 18$	19	20	21
β	$2 \vee \neg 3$	α		β	γ		ζ	η	κ		ν		d	δ	θ	ξ	λ	ι	\omicron	π	ρ	ϵ
γ	$2 \vee \neg 4$																					
δ	$3 \vee \neg 12 \vee \neg 13$																					
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$																					
ζ	$\neg 5 \vee \neg 6$																					
η	$6 \vee 7$																					
θ	$6 \vee \neg 12 \vee 14$																					
ι	$\neg 7 \vee 16 \vee 17$																					
κ	$\neg 8 \vee 9$																					
λ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$																					
μ	$\neg 9 \vee \neg 19 \vee \neg 21$																					
ν	$10 \vee 11$																					
ξ	$13 \vee \neg 14 \vee \neg 15$																					
\omicron	$16 \vee \neg 18$																					
π	$16 \vee 19$																					
ρ	$\neg 19 \vee 20$																					



$\neg 9 \vee \neg 19 \vee \neg 21$ conflict clause (μ)
 resolve with ϵ

Example

α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	$\neg 13$	14	$\neg 15$	$\neg 16$	17	$\neg 18$	19	20	21
β	$2 \vee \neg 3$	α	β	γ	ζ	η	κ	ν	δ	θ	ξ	λ	ι	\omicron	π	ρ	ϵ					
γ	$2 \vee \neg 4$																					
δ	$3 \vee \neg 12 \vee \neg 13$																					
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$																					
ζ	$\neg 5 \vee \neg 6$																					
η	$6 \vee 7$																					
θ	$6 \vee \neg 12 \vee 14$																					
ι	$\neg 7 \vee 16 \vee 17$																					
κ	$\neg 8 \vee 9$																					
λ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$																					
μ	$\neg 9 \vee \neg 19 \vee \neg 21$																					
ν	$10 \vee 11$																					
ξ	$13 \vee \neg 14 \vee \neg 15$																					
\omicron	$16 \vee \neg 18$																					
π	$16 \vee 19$																					
ρ	$\neg 19 \vee 20$																					

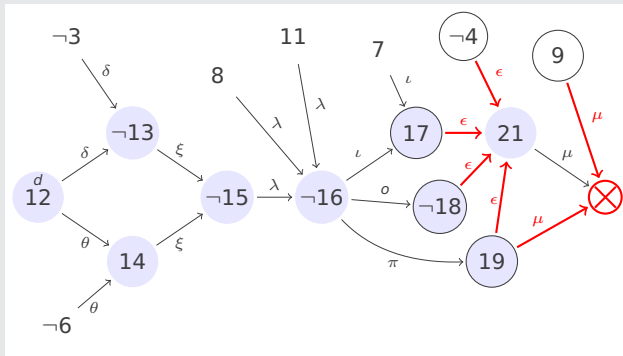


$\neg 9 \vee \neg 19 \vee \neg 21$ conflict clause (μ)

$4 \vee \neg 9 \vee \neg 17 \vee 18 \vee \neg 19$ resolve with ϵ

Example

α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	$\neg 13$	14	$\neg 15$	$\neg 16$	17	$\neg 18$	19	20	21
β		2	\vee	$\neg 3$																		
γ		2	\vee	$\neg 4$																		
δ		3	\vee	$\neg 12$	\vee	$\neg 13$																
ϵ		4	\vee	$\neg 17$	\vee	18	\vee	$\neg 19$	\vee	21												
ζ		$\neg 5$	\vee	$\neg 6$																		
η		6	\vee	7																		
θ		6	\vee	$\neg 12$	\vee	14																
ι		$\neg 7$	\vee	16	\vee	17																
κ		$\neg 8$	\vee	9																		
λ		$\neg 8$	\vee	$\neg 11$	\vee	15	\vee	$\neg 16$														
μ		$\neg 9$	\vee	$\neg 19$	\vee	$\neg 21$																
ν		10	\vee	11																		
ξ		13	\vee	$\neg 14$	\vee	$\neg 15$																
\omicron		16	\vee	$\neg 18$																		
π		16	\vee	19																		
ρ		$\neg 19$	\vee	20																		

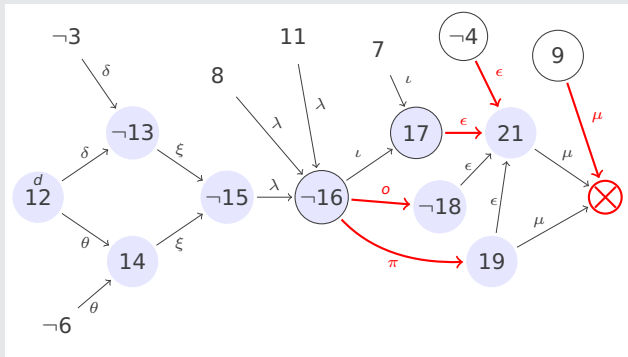


$\neg 9 \vee \neg 19 \vee \neg 21$ conflict clause (μ)

$4 \vee \neg 9 \vee \neg 17 \vee 18 \vee \neg 19$ resolve with ϵ, π

Example

α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	$\neg 13$	14	$\neg 15$	$\neg 16$	17	$\neg 18$	19	20	21
β	$2 \vee \neg 3$	α	β	γ	ζ	η	κ	ν	δ	θ	ξ	λ	ι	\circ	π	ρ	ϵ					
γ	$2 \vee \neg 4$																					
δ	$3 \vee \neg 12 \vee \neg 13$																					
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$																					
ζ	$\neg 5 \vee \neg 6$																					
η	$6 \vee 7$																					
θ	$6 \vee \neg 12 \vee 14$																					
ι	$\neg 7 \vee 16 \vee 17$																					
κ	$\neg 8 \vee 9$																					
λ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$																					
μ	$\neg 9 \vee \neg 19 \vee \neg 21$																					
ν	$10 \vee 11$																					
ξ	$13 \vee \neg 14 \vee \neg 15$																					
\circ	$16 \vee \neg 18$																					
π	$16 \vee 19$																					
ρ	$\neg 19 \vee 20$																					

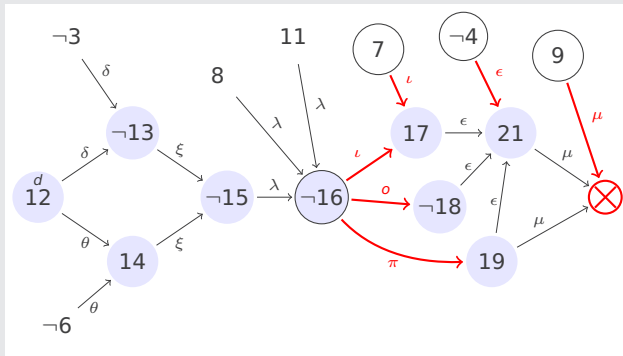


$\neg 9 \vee \neg 19 \vee \neg 21$ conflict clause (μ)

$4 \vee \neg 9 \vee 16 \vee \neg 17$ resolve with ϵ , π , \circ

Example

α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	$\neg 13$	14	$\neg 15$	$\neg 16$	17	$\neg 18$	19	20	21
β	$2 \vee \neg 3$	α		β	γ		ζ	η	κ		ν			δ	θ	ξ	λ	ι	\omicron	π	ρ	ϵ
γ	$2 \vee \neg 4$																					
δ	$3 \vee \neg 12 \vee \neg 13$																					
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$																					
ζ	$\neg 5 \vee \neg 6$																					
η	$6 \vee 7$																					
θ	$6 \vee \neg 12 \vee 14$																					
ι	$\neg 7 \vee 16 \vee 17$																					
κ	$\neg 8 \vee 9$																					
λ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$																					
μ	$\neg 9 \vee \neg 19 \vee \neg 21$																					
ν	$10 \vee 11$																					
ξ	$13 \vee \neg 14 \vee \neg 15$																					
\omicron	$16 \vee \neg 18$																					
π	$16 \vee 19$																					
ρ	$\neg 19 \vee 20$																					

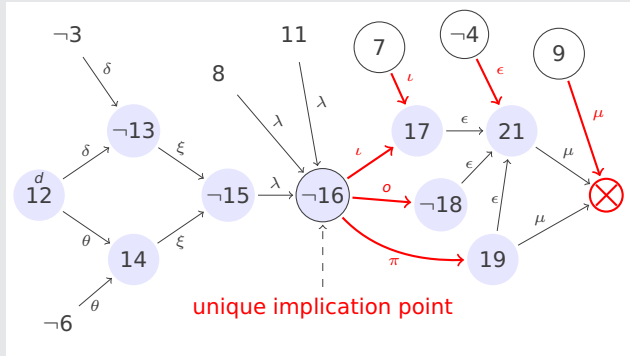


$\neg 9 \vee \neg 19 \vee \neg 21$ conflict clause (μ)

$4 \vee \neg 7 \vee \neg 9 \vee 16$ resolve with ϵ , π , \omicron , ι

Example

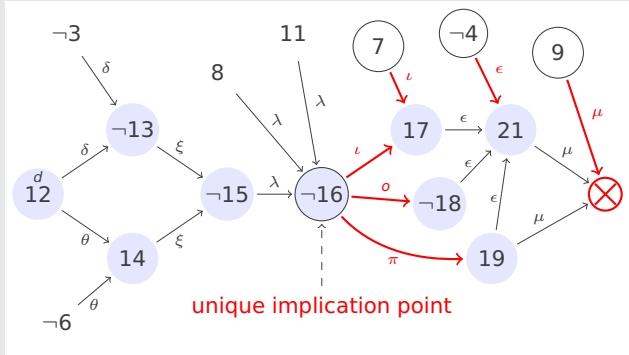
α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	$\neg 13$	14	$\neg 15$	$\neg 16$	17	$\neg 18$	19	20	21
β	$2 \vee \neg 3$	α	β	γ	ζ	η	κ	ν	δ	θ	ξ	λ	ι	ϵ	ρ	σ	π	ρ	ϵ			
γ	$2 \vee \neg 4$																					
δ	$3 \vee \neg 12 \vee \neg 13$																					
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$																					
ζ	$\neg 5 \vee \neg 6$																					
η	$6 \vee 7$																					
θ	$6 \vee \neg 12 \vee 14$																					
ι	$\neg 7 \vee 16 \vee 17$																					
κ	$\neg 8 \vee 9$																					
λ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$																					
μ	$\neg 9 \vee \neg 19 \vee \neg 21$																					
ν	$10 \vee 11$																					
ξ	$13 \vee \neg 14 \vee \neg 15$																					
σ	$16 \vee \neg 18$																					
π	$16 \vee 19$																					
ρ	$\neg 19 \vee 20$																					



$\neg 9 \vee \neg 19 \vee \neg 21$ conflict clause (μ)
 $4 \vee \neg 7 \vee \neg 9 \vee 16$ resolve with $\epsilon, \pi, \sigma, \iota$

Example

α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	$\neg 13$	14	$\neg 15$	$\neg 16$	17	$\neg 18$	19	20	21
β	$2 \vee \neg 3$	α	β	γ	ζ	η	κ	ν	δ	θ	ξ	λ	ι	ϵ	ρ	σ	π	μ	ν	ξ	ρ	ϵ
γ	$2 \vee \neg 4$																					
δ	$3 \vee \neg 12 \vee \neg 13$																					
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$																					
ζ	$\neg 5 \vee \neg 6$																					
η	$6 \vee 7$																					
θ	$6 \vee \neg 12 \vee 14$																					
ι	$\neg 7 \vee 16 \vee 17$																					
κ	$\neg 8 \vee 9$																					
λ	$\neg 8 \vee \neg 11 \vee 15 \vee \neg 16$																					
μ	$\neg 9 \vee \neg 19 \vee \neg 21$																					
ν	$10 \vee 11$																					
ξ	$13 \vee \neg 14 \vee \neg 15$																					
σ	$16 \vee \neg 18$																					
π	$16 \vee 19$																					
ρ	$\neg 19 \vee 20$																					



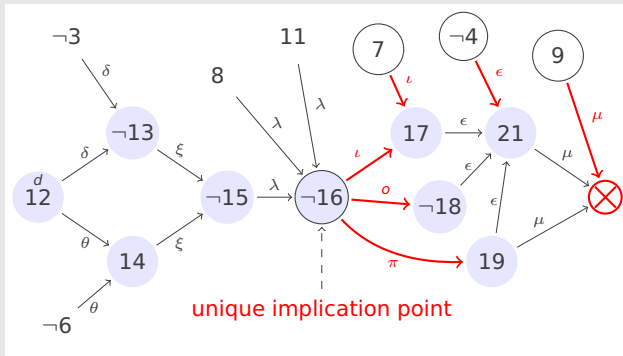
$\neg 9 \vee \neg 19 \vee \neg 21$ conflict clause (μ)

$4 \vee \neg 7 \vee \neg 9 \vee 16$ resolve with $\epsilon, \pi, \sigma, \iota$

UIP: exactly one literal at current decision level

Example

α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	$\neg 13$	14	$\neg 15$	$\neg 16$	17	$\neg 18$	19	20	21
β		2	\vee	$\neg 3$																		
γ		2	\vee	$\neg 4$																		
δ		3	\vee	$\neg 12$	\vee	$\neg 13$																
ϵ		4	\vee	$\neg 17$	\vee	18	\vee	$\neg 19$	\vee	21												
ζ		$\neg 5$	\vee	$\neg 6$																		
η		6	\vee	7																		
θ		6	\vee	$\neg 12$	\vee	14																
ι		$\neg 7$	\vee	16	\vee	17																
κ		$\neg 8$	\vee	9																		
λ		$\neg 8$	\vee	$\neg 11$	\vee	15	\vee	$\neg 16$														
μ		$\neg 9$	\vee	$\neg 19$	\vee	$\neg 21$																
ν		10	\vee	11																		
ξ		13	\vee	$\neg 14$	\vee	$\neg 15$																
\omicron		16	\vee	$\neg 18$																		
π		16	\vee	19																		
ρ		$\neg 19$	\vee	20																		



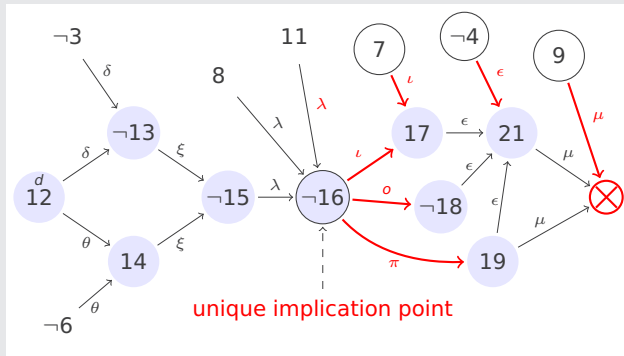
$\neg 9 \vee \neg 19 \vee \neg 21$ conflict clause (μ)

$4 \vee \neg 7 \vee \neg 9 \vee 16$ resolve with $\epsilon, \pi, \omicron, \iota$

backjump clause

Example

α 1 1 $\overset{d}{\neg 2}$ $\neg 3$ $\neg 4$ $\overset{d}{5}$ $\neg 6$ $\overset{d}{7}$ $\overset{d}{8}$ $\overset{d}{9}$ **16** **backjump**
 β 2 \vee $\neg 3$
 γ 2 \vee $\neg 4$
 δ 3 \vee $\neg 12$ \vee $\neg 13$
 ϵ 4 \vee $\neg 17$ \vee 18 \vee $\neg 19$ \vee **21**
 ζ $\neg 5$ \vee $\neg 6$
 η 6 \vee 7
 θ 6 \vee $\neg 12$ \vee **14**
 ι $\neg 7$ \vee 16 \vee **17**
 κ $\neg 8$ \vee **9**
 λ $\neg 8$ \vee $\neg 11$ \vee 15 \vee $\neg 16$
 μ $\neg 9$ \vee $\neg 19$ \vee $\neg 21$
 ν 10 \vee **11**
 ξ 13 \vee $\neg 14$ \vee $\neg 15$
 \omicron 16 \vee $\neg 18$
 π 16 \vee **19**
 ρ $\neg 19$ \vee **20**

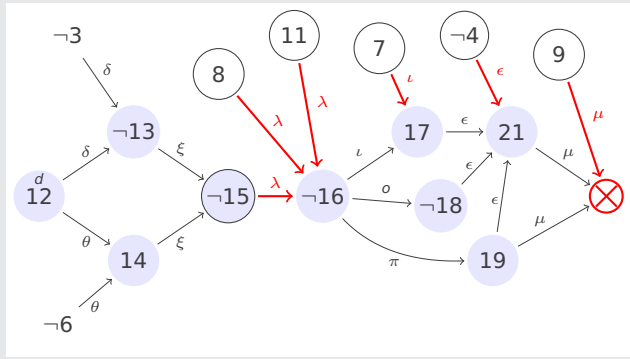


$\neg 9 \vee \neg 19 \vee \neg 21$ conflict clause (μ)
 $4 \vee \neg 7 \vee \neg 9 \vee 16$ resolve with $\epsilon, \pi, \omicron, \iota$

backjump clause

Example

α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	$\neg 13$	14	$\neg 15$	$\neg 16$	17	$\neg 18$	19	20	21
β	$2 \vee \neg 3$																					
γ	$2 \vee \neg 4$																					
δ	$3 \vee \neg 12 \vee \neg 13$																					
ϵ	$4 \vee \neg 17 \vee 18 \vee \neg 19 \vee 21$																					
ζ	$\neg 5 \vee \neg 6$																					
η	$6 \vee 7$																					
θ	$6 \vee \neg 12 \vee 14$																					
ι	$\neg 7 \vee 16 \vee 17$																					
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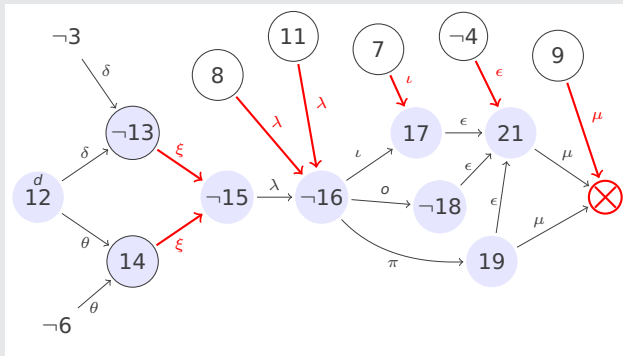


$\neg 9 \vee \neg 19 \vee \neg 21$ conflict clause (μ)

$4 \vee \neg 7 \vee \neg 8 \vee \neg 9 \vee \neg 11 \vee 15$ resolve with $\epsilon, \pi, \omicron, \iota, \lambda$

Example

α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	$\neg 13$	14	$\neg 15$	$\neg 16$	17	$\neg 18$	19	20	21
β	$2 \vee \neg 3$	α		β	γ		ζ	η	κ		ν			δ	θ	ξ	λ	ι	\omicron	π	ρ	ϵ
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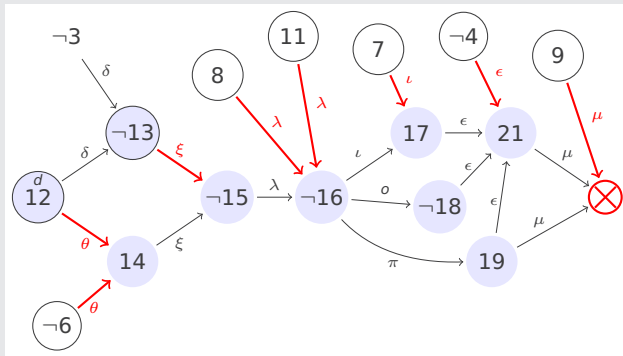


$\neg 9 \vee \neg 19 \vee \neg 21$ conflict clause (μ)

$4 \vee \neg 7 \vee \neg 8 \vee \neg 9 \vee \neg 11 \vee 13 \vee \neg 14$ resolve with $\epsilon, \pi, \omicron, \iota, \lambda, \xi$

Example

α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	$\neg 13$	14	$\neg 15$	$\neg 16$	17	$\neg 18$	19	20	21
β	$2 \vee \neg 3$	α	β	γ	ζ	η	κ	ν	δ	θ	ξ	ζ	λ	ι	\omicron	π	ρ	ϵ				
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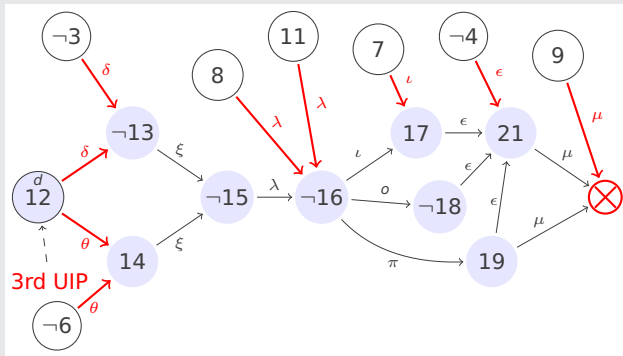


$\neg 9 \vee \neg 19 \vee \neg 21$ conflict clause (μ)

$4 \vee 6 \vee \neg 7 \vee \neg 8 \vee \neg 9 \vee \neg 11 \vee \neg 12 \vee 13$ resolve with $\epsilon, \pi, \omicron, \iota, \lambda, \xi, \theta$

Example

α	1	1	$\neg 2$	$\neg 3$	$\neg 4$	5	$\neg 6$	7	8	9	$\neg 10$	11	12	$\neg 13$	14	$\neg 15$	$\neg 16$	17	$\neg 18$	19	20	21
β			\vee	$\neg 3$																		
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δ			3	\vee	$\neg 12$	\vee	$\neg 13$															
ϵ			4	\vee	$\neg 17$	\vee	18	\vee	$\neg 19$	\vee	21											
ζ			$\neg 5$	\vee	$\neg 6$																	
η			6	\vee	7																	
θ			6	\vee	$\neg 12$	\vee	14															
ι			$\neg 7$	\vee	16	\vee	17															
κ			$\neg 8$	\vee	9																	
λ			$\neg 8$	\vee	$\neg 11$	\vee	15	\vee	$\neg 16$													
μ			$\neg 9$	\vee	$\neg 19$	\vee	$\neg 21$															
ν			10	\vee	11																	
ξ			13	\vee	$\neg 14$	\vee	$\neg 15$															
\omicron			16	\vee	$\neg 18$																	
π			16	\vee	19																	
ρ			$\neg 19$	\vee	20																	



$\neg 9 \vee \neg 19 \vee \neg 21$ conflict clause (μ)

$3 \vee 4 \vee 6 \vee \neg 7 \vee \neg 8 \vee \neg 9 \vee \neg 11 \vee \neg 12$ resolve with $\epsilon, \pi, \omicron, \iota, \lambda, \xi, \theta, \delta$

Remarks

- ▶ computed clauses are clauses that correspond to **cut** in conflict graph

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Observation

adding backjump clauses to clause database (**learning**) helps to prune search space

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Observation

adding backjump clauses to clause database (learning) helps to prune search space

▶ learn

$$M \parallel F \implies M \parallel F, C$$

if $F \models C$ and each atom of C occurs in F or in M

Observation

restarts are useful to avoid wasting too much time in parts of search space without satisfying assignments

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► restart

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Final Remarks

▶ restarts do not compromise completeness if number of steps between consecutive restarts strictly increases

Observation

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$$M \parallel F \implies \parallel F$$

Final Remarks

- ▶ restarts do not compromise completeness if number of steps between consecutive restarts strictly increases
- ▶ modern SAT solvers additionally incorporate
 - ▶ heuristics for selecting next decision literal

Observation

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Final Remarks

- ▶ restarts do not compromise completeness if number of steps between consecutive restarts strictly increases
- ▶ modern SAT solvers additionally incorporate
 - ▶ heuristics for selecting next decision literal
 - ▶ special data structures that allow for efficient unit propagation

Outline

1. Summary of Previous Lecture
2. Sorting Networks
3. Intermezzo
4. Conflict Graph
- 5. Further Reading**
6. Course Recommendations
7. Exam

- ▶ Section 3.5

DPLL

- ▶ Section 2 of Solving SAT and SAT Modulo Theories: From an Abstract Davis–Putnam–Logemann–Loveland Procedure to DPLL(T)

Robert Nieuwenhuis, Albert Oliveras, and Cesare Tinelli

Journal of the ACM 53(6), pp. 937–977, 2006

doi: [10.1145/1217856.1217859](https://doi.org/10.1145/1217856.1217859)

- ▶ Conflict-Driven Clause Learning SAT Solvers

Joao Marques–Silva, Ines Lynce, and Sharad Mali

Chapter 4 of Handbook of Satisfiability, IOS Press, 2008

www.ics.uci.edu/~dechter/courses/ics-275a/winter-2016/readings/SATHandbook-CDCL.pdf

- ▶ The Silent (R)evolution of SAT

Johannes K. Fichte, Daniel Le Berre, Markus Hecher, and Stefan Szeider

Communications of the ACM 66(6), pp. 64–72, 2023

doi: [10.1145/3560469](https://doi.org/10.1145/3560469)

SAT

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Sorting Networks

- ▶ Section 5.3.4 of The Art of Computer Programming

Donald Knuth

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Important Concepts

- ▶ bitonic sorting
- ▶ conflict graph
- ▶ cut
- ▶ learn
- ▶ odd-even mergesort
- ▶ restart
- ▶ unit propagation
- ▶ unique implication point (UIP)
- ▶ zero-one principle

homework for June 20

Grading — Proseminar

$$\text{score} = \min\left(\frac{50}{67}(E + P) + B, 100\right)$$

Grading — Proseminar

$$\text{score} = \min\left(\frac{50}{67}(E + P) + B, 100\right)$$

grade : $[0, 50) \rightarrow \mathbf{5}$ $[50, 63) \rightarrow \mathbf{4}$ $[63, 75) \rightarrow \mathbf{3}$ $[75, 88) \rightarrow \mathbf{2}$ $[88, 100] \rightarrow \mathbf{1}$

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Courses in 2024/2025

▶ Program Verification	(WM 1)	VO3 + PS2	SS 2025
▶ Term Rewriting	(PM 20)	VU3	SS 2025

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▶ Program Verification	(WM 1)	VO3 + PS2	SS 2025
▶ Term Rewriting	(PM 20)	VU3	SS 2025
▶ Automata and Logic	(MSc WM 1)	VO2 + PS2	WS 2024
▶ Constraint Solving	(MSc WM 2)	VO2 + PS2	SS 2025

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▶ Term Rewriting	(PM 20)	VU3	SS 2025
▶ Automata and Logic	(MSc WM 1)	VO2 + PS2	WS 2024
▶ Constraint Solving	(MSc WM 2)	VO2 + PS2	SS 2025
▶ Program and Resource Analysis	(MSc WM 8)	VU3	WS 2024
▶ Tree Automata	(MSc WM 9)	VU3	WS 2024

Courses in 2024/2025

▶ Program Verification	(WM 1)	VO3 + PS2	SS 2025
▶ Term Rewriting	(PM 20)	VU3	SS 2025
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▶ Constraint Solving	(MSc WM 2)	VO2 + PS2	SS 2025
▶ Program and Resource Analysis	(MSc WM 8)	VU3	WS 2024
▶ Tree Automata	(MSc WM 9)	VU3	WS 2024
▶ Semantics of Programming Languages	(MSc WM 7)	VU3	SS 2025
▶ Quantum Computation	(MSc WM 8)	VU3	SS 2025

Courses in 2024/2025

▶ Program Verification	(WM 1)	VO3 + PS2	SS 2025
▶ Term Rewriting	(PM 20)	VU3	SS 2025
▶ Automata and Logic	(MSc WM 1)	VO2 + PS2	WS 2024
▶ Constraint Solving	(MSc WM 2)	VO2 + PS2	SS 2025
▶ Program and Resource Analysis	(MSc WM 8)	VU3	WS 2024
▶ Tree Automata	(MSc WM 9)	VU3	WS 2024
▶ Semantics of Programming Languages	(MSc WM 7)	VU3	SS 2025
▶ Quantum Computation	(MSc WM 8)	VU3	SS 2025
▶ Term Rewriting	(PM 20)	VU2	(Obergurgl)
▶ Lambda Calculus and Type Theory	(PM 20)	VU2	(Obergurgl)

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▶ Automata and Logic	(MSc WM 1)	VO2 + PS2	WS 2024
▶ Constraint Solving	(MSc WM 2)	VO2 + PS2	SS 2025
▶ Program and Resource Analysis	(MSc WM 8)	VU3	WS 2024
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bachelor projects

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First Exam on June 24

- ▶ **8:30** – 11:00 in HSB 3 and HSB 1
- ▶ deregistration is possible until 23:59 on June 20
- ▶ closed book
- ▶ second exam on September 20, third exam on February 26, 2025

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Preparation

- ▶ study **previous exams**
- ▶ review homework exercises and solutions
- ▶ study slides
- ▶ visit Tutorium Wednesday, 16:15 – 17:00, SR 13
- ▶ visit consultation hours AM Wednesday, 11:30 – 13:00, 3M07

evaluation SS 2024