universität innsbruck

Constraint Solving | SS 2025 | LVA 703304

EXAM 1 | June 24, 2025

This exam consists of **6** exercises. The available points for each item are written in the margin. In total there are 90 points. You need 45 points to pass.

1 Algorithms for Linear Arithmetic

Consider the following formula:

$$\varphi := \exists x.\exists y.\ 2x + 3y < 1 \wedge x + y \geq 5$$

[10] (a) Remove the quantifier of y within φ using the algorithm of Ferrante and Rackoff. You *do not* have to simplify the formula after the removal of the quantifier.

[10] (b) Start to solve φ using the simplex method: apply all initial steps and *one* iteration of the main loop. Use Bland's selection rule with the variable order $y < x < t < s$ where s and t are the introduced slack variables.
Is a second iteration of the main loop required? Just answer this question with a yes or no.

2 Algorithms for Difference Logic

[1] (a) The theory solver for difference logic is using a shortest path algorithm. Specify whether this algorithm is the one of Bellman and Ford, the one of Dijkstra, or the one of Floyd and Warshall.

(b) In the shortest path algorithm for difference logic, the array with the computed distances is updated in $|V| - 1$ many iterations, where V is the set of vertices of the graph. Now consider the situation where the algorithm is modified in such a way that it only performs $|V| - 2$ many iterations.

For each of the following problematic situations, argue that it cannot happen, or provide an example graph where such a situation can arise.

[3] i. The modified algorithm reports a negative cycle, although the graph has no negative cycle.

[3] ii. The modified algorithm returns a distance array, the graph has no negative cycle, but the distances are incorrect.

[3] iii. The modified algorithm returns a distance array, but the graph has a negative cycle.

3 Algorithms for Equation Handling

[10] Apply Griggio's algorithm to convert E into an equi-satisfiable set of equations S that is in solved form. Here, E consists of the following two equations.

$$3x + 12y - 6z = 15$$
$$\frac{1}{2}x + 3y + \frac{3}{2}z = 2$$

Provide intermediate results and explain your calculation.

4 Algorithms for Combinations of Theories

[10] (a) Consider the EUF+LIA formula ψ over variables $\{x, y, z\}$ defined as:

$$f(x+1) = g(y) \wedge x < z \leq x+2 \longrightarrow f(z) = g(y)$$

Investigate *validity* of ψ with the help of the Nelson–Oppen algorithm.

- Briefly mention the applied steps and write down intermediate results
- Mention each invocation of an SMT-solver for LIA or of an SMT-solver for EUF. Just mention the (obvious) results of these invocations, and do *not* provide a detailed trace of the execution of the SMT-solvers.

[10] (b) SMT solving via the non-deterministic Nelson–Oppen algorithm was introduced for arbitrary quantifier-free formulas φ over two stably infinite theories that only share the equality predicate. With these assumption, the Nelson–Oppen algorithm is a decision procedure for satisfiability.

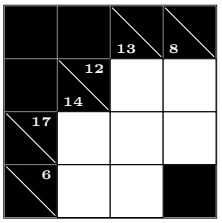
Investigate what happens if one drops the assumption of stably infinite theories. Is the following statement still true?

- If the Nelson–Oppen algorithm on formula φ returns "unsat", then φ is indeed not satisfiable.

Provide a counter-example or briefly argue why the statement is still satisfied.

5 Encoding Problems

Consider the following Kakuro puzzle.



[10] Encode this puzzle in LRA. To remind you, here are the rules:

(a) Every white cell is filled with a digit between 1 and 9.

(b) Every row and every line must be filled with distinct digits.

(c) The numbers indicate the required sum of the digits in that line or row.

Please indicate which parts of the encoding take care of which aspects of the rules; moreover, describe the meaning of the variables.

6 Multiple Choice

[20] There are ten questions on the answer sheet.

Mark your answers by crossing the correct box, e.g., like this: ☑.

- Each correct answer is worth 2 points.
- Each wrong answer is worth -1 point.
- Giving no answer to a question is worth 0 points.
- If the total number of points is negative, then this exercise will be evaluated with 0 points.