This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass. ***Explain your answers to the first four exercises!***

[7] **1** (a) Use DPLL to check whether the formula φ:

$$(r \vee q \vee p) \wedge (\neg p \vee s \vee r) \wedge (\neg r \vee \neg p) \wedge (\neg r \vee \neg q \vee \neg p) \wedge (\neg r \vee p) \wedge (\neg s \vee r \vee \neg q)$$

is satisfiable.

[7] (b) Use Tseitin's transformation to turn the formula $\psi = \neg(p \wedge q) \rightarrow (\neg q \vee r)$ into an equisatisfiable CNF.

[6] (c) Consider the propositional formula χ:

$$(q \rightarrow s) \wedge (s \wedge t \rightarrow \bot) \wedge (\top \rightarrow q) \wedge (s \wedge r \rightarrow t) \wedge (q \rightarrow r) \wedge (r \rightarrow s)$$

i. Is χ a Horn formula? If not, why? If yes, is it satisfiable?

ii. Transform χ into an equivalent CNF. Is χ valid?

[6] **2** (a) Use resolution to determine satisfiability of the clausal form

$$\{\{\neg P(x), Q(x)\}, \{\neg Q(a)\}, \{P(b), R(x,y)\}, \{S(x), \neg R(a,b)\}, \{\neg S(a)\}\}$$

where a and b are constants.

[7] (b) Compute a most general unifier of the terms

$$g(f(x, f(b,a)), g(x, f(b,y))) \quad \text{and} \quad g(f(g(y,b), z), g(g(a,b), z))$$

or argue why this is not possible. Here, a and b are constants and f and g are binary functions.

[7] (c) Transform the following formula into an equisatisfiable Skolem normal form:

$$\varphi = \forall x\, \exists y\, \forall z\, \big(R(x,y,z) \rightarrow \exists w\, S(y,z,w)\big)$$

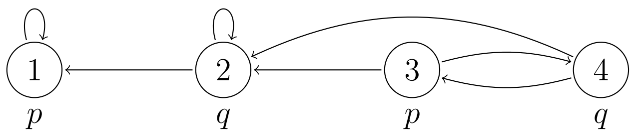
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it. In parts (b) and (c), a and b are constants and P and Q are unary predicate symbols.

[6] (a) $p,\ r \rightarrow \neg p,\ \neg r \wedge s \rightarrow t,\ \neg t \vdash \neg s$

[7] (b) $\forall x\,(x = a \vee x = b),\ \forall x\,(x = a \rightarrow P(x)),\ \forall x\,(x = b \rightarrow Q(x)) \vdash \forall x\,(P(x) \vee Q(x))$

[7] (c) $\forall x\,(x = a \vee x = b),\ \forall x\,(x = a \rightarrow P(x)),\ \forall x\,(x = b \rightarrow Q(x)) \vdash \forall x\, P(x) \vee \forall x\, Q(x)$

[4] Consider the following model $\mathcal{M}$:



[6] (a) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}$ the CTL formula $\varphi = \mathsf{EF}\ \mathsf{A}[\mathsf{EF}\ p\ \mathsf{U}\ \mathsf{AX}\ q]$ holds.

[7] (b) Give an LTL formula ψ such that $\mathcal{M}, s \models \psi$ if and only if $s = 2$.

[7] (c) Show that the two CTL formulas with fairness constraints $\mathsf{E}_{\{p,q\}}\ \mathsf{G}\top$ and $\mathsf{E}_{\{p \wedge q\}}\ \mathsf{G}\top$ are not equivalent.

[20] [5] Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
Austria is 3-colorable.
The formulas $p \wedge \neg q$ and $p \wedge q$ are equisatifiable.
The proof rule $\neg\neg$e is a derived rule of natural deduction.
The clause $\{P(z)\}$ is a resolvent of $\{\neg P(x)\}$ and $\{P(z), P(y)\}$.
If a binary function f is monotone and $f(1,1) = 0$ then $f(x,y) = 0$.
The set $\{\mathsf{R}, \mathsf{U}, \mathsf{X}\}$ is an adequate set of temporal connectives for LTL.
Deciding the satisfiability of propositional Horn formulas is NP-complete.
The problem whether an arbitrary propositional formula is valid is decidable.
The sequent $\forall x\, \exists y\, P(x,y), \forall x\, \forall y\, (P(x,y) \rightarrow Q(x,y)) \vdash \exists y\, \forall x\, Q(x,y)$ is valid.
Every reduced OBDD for an n-ary boolean function has at most $2^{n+1} - 1$ nodes.