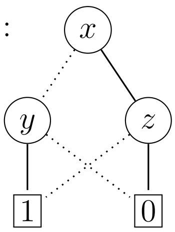
universität innsbruck

Logik	25S	LVA 703026
EXAM 3		February 5, 2026

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass. ***Explain your answers to the first four exercises!***

1 Consider the boolean function f defined by $f(x, y, z) = x \oplus (\overline{y} + z)$ and the BDD B_g

B_g:



[6] (a) Compute a reduced OBDD for f with variable ordering $[x, y, z]$.

[7] (b) Starting from B_g, compute a reduced OBDD that is equivalent to $\exists y.g$.

[7] (c) Which subsets of $\{f, g\}$ are adequate?

[6] 2 (a) Compute a most general unifier of the terms

$$f(g(f(z, b), a, f(b, y)), f(y, x)) \quad \text{and} \quad f(g(y, z, f(x, f(a, x))), v)$$

or argue why this is not possible. Here, a and b are constants, f is a binary function, g is a ternary function, and v, x, y and z are variables.

[7] (b) Use resolution to determine satisfiablity of the following CNF:

$$(p \vee \neg q) \wedge (r \vee p \vee \neg s) \wedge (\neg r \vee \neg p) \wedge (\neg s \vee q)$$

[7] (c) Transform the following formula into an equisatisfiable Skolem normal form:

$$\exists x\, (\forall y\, (P(x) \to Q(y, x))) \to \forall z\, P(z)$$

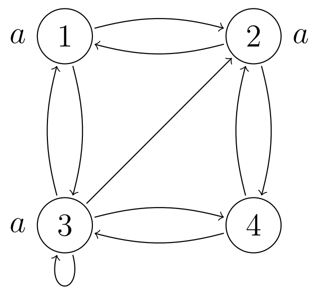
3 For each of the following sequents, either give a natural deduction proof or find a model which does not satisfy it.

[6] (a) $\neg(p \wedge q) \vdash \neg p \vee \neg q$

[7] (b) $\vdash \forall x\, \exists y\, (P(x) \to Q(y)) \to \forall x\, (P(x) \to \exists y\, Q(y))$

[7] (c) $\vdash \forall x\, \forall y\, (R(x, y) \to (\exists z\, (R(x, z) \wedge R(z, y))))$

4 Consider the following model $\mathcal{M}$:



[6] (a) Use the CTL model checking algorithm to determine in which states of $\mathcal{M}$ the CTL formula $\varphi = \mathsf{AF}\,\mathsf{E}[\mathsf{AX}\,a\,\mathsf{U}\,\mathsf{EX}\,\neg a]$ holds.

[8] (b) For each $1 \leqslant i \leqslant 4$ construct a CTL formula ψ_i which holds only in state i of $\mathcal{M}$.

[6] (c) Find an LTL formula χ such that neither $\mathcal{M}, 2 \vDash \chi$ nor $\mathcal{M}, 2 \vDash \neg\chi$.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
The set $\{\mathsf{EX}, \mathsf{EU}, \mathsf{AF}\}$ is adequate for CTL.
The formulas $(p \vee q) \wedge \neg p$ and $\top$ are equisatisfiable.
Resolution is sound and complete for predicate logic.
Intuitionistic logicans do not use LEM, PBC and $\rightarrow$e.
Deciding the satisfiability of CNF formulas is NP-complete.
The formula $(p \wedge q \rightarrow s) \wedge (s \rightarrow r) \wedge (q \rightarrow \bot)$ is a Horn formula.
Every boolean function has a unique representation as reduced BDD.
The set $[\![\mathsf{AF}\,\varphi]\!]$ is the least fixed point of function $F_{\mathsf{AF}}(X) = [\![\varphi]\!] \cap \mathsf{pre}_\forall(X)$.
The sequent $\exists x\,\exists y\,(P(x,y) \vee P(y,x)), \neg\exists x\,P(x,x) \vdash \exists x\,\exists y\,\neg(x = y)$ is valid.
An n-ary boolean function f is not self-dual if and only if $f(b_1,\dots,b_n) = f(\overline{b_1},\dots,\overline{b_n})$ for all $b_1,\dots,b_n \in \{0,1\}$.