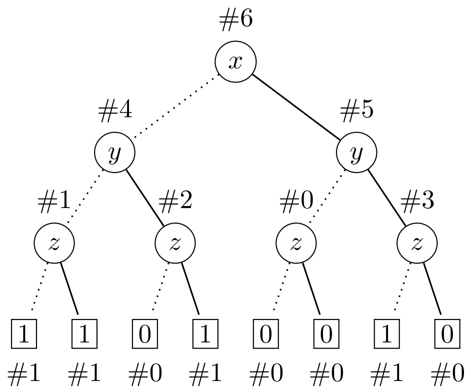
1 (a) *answer + explanation*

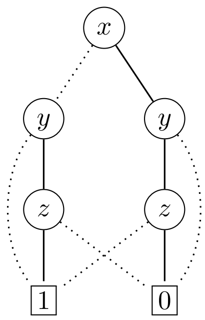
From the table

x	y	z	$f(x,y,z)$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

we obtain the binary decision tree

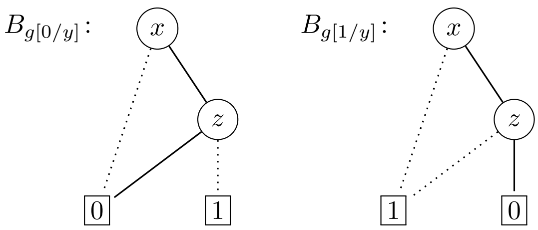


Applying the reduce algorithm produces the desired reduced OBDD:

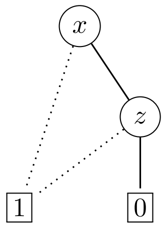


(b) *answer + explanation*

Applying restrict yields reduced OBDDs for $B_{g[0/y]}$ and $B_{g[1/y]}$:



Computing apply$(+, B_{g[0/y]}, B_{g[1/y]})$ yields the reduced OBDD for $\exists y.f$



which is equal to $B_{g[1/y]}$.

(c) *answer + explanation*

We have $g(0,0,0) = g(1,1,1) = f(1,1,1) = 0$ and $f(0,0,0) = 1$. Neither f nor g is monotone: $f(0,0,0) = 1 > 0 = f(0,1,0)$ and $g(1,1,0) = 1 > 0 = g(1,1,1)$. Moreover, $f(0,1,0) = 0 = f(1,0,1)$ and $g(0,0,0) = 0 = g(1,1,1)$, so f and g are not self-dual. The ANF of f is $1 \oplus x \oplus y \oplus yz$ and the ANF of g is $x \oplus y \oplus xy \oplus xz$. Both are not linear, so f and g are not affine. The following table summarizes our findings:

	f	g
$h(0,\cdots,0) \neq 0$	✓	
$h(1,\cdots,1) \neq 1$	✓	✓
not monotone	✓	✓
not self-dual	✓	✓
not affine	✓	✓

Hence, $\{f, g\}$ and $\{f\}$ are the adequate subsets of $\{f, g\}$.

2 (a)

answer + computation

The terms are unifiable:

$$
\begin{array}{c}
f(g(f(z,b),a,f(b,y)),f(y,x)) \approx f(g(y,z,f(x,f(a,x))),v) \\
\mathsf{d} \Downarrow \\
g(f(z,b),a,f(b,y)) \approx g(y,z,f(x,f(a,x))),\ f(y,x) \approx v \\
\mathsf{d} \Downarrow \\
f(z,b) \approx y,\ a \approx z,\ f(b,y) \approx f(x,f(a,x)),\ f(y,x) \approx v \\
\mathsf{v} \Downarrow \{y \mapsto f(z,b)\} \\
a \approx z,\ f(b,f(z,b)) \approx f(x,f(a,x)),\ f(f(z,b),x) \approx v \\
\mathsf{v} \Downarrow \{z \mapsto a\} \\
f(b,f(a,b)) \approx f(x,f(a,x)),\ f(f(a,b),x) \approx v \\
\mathsf{d} \Downarrow \\
b \approx x, f(a,b) \approx f(a,x),\ f(f(a,b),x) \approx v \\
\mathsf{v} \Downarrow \{x \mapsto b\} \\
f(a,b) \approx f(a,b),\ f(f(a,b),b) \approx v \\
\mathsf{t} \Downarrow \\
f(f(a,b),b) \approx v \\
\mathsf{v} \Downarrow \{v \mapsto f(f(a,b),b)\} \\
\square
\end{array}
$$

The resulting mgu is

$$
\begin{aligned}
&\{y \mapsto f(z,b)\}\{z \mapsto a\}\{x \mapsto b\}\{v \mapsto f(f(a,b),b)\} \\
&= \{v \mapsto f(f(a,b),b),\ x \mapsto b,\ y \mapsto f(a,b),\ z \mapsto a\}
\end{aligned}
$$

(b) *answer + explanation*

Resolution produces the following clauses:

1.	$\{p, \neg q\}$	
2.	$\{p, r, \neg s\}$	
3.	$\{\neg p, \neg r\}$	
4.	$\{q, \neg s\}$	
5.	$\{\neg q, \neg r\}$	resolve 1, 3, p
6.	$\{p, \neg s\}$	resolve 1, 4, q
7.	$\{r, \neg r, \neg s\}$	resolve 2, 3, p
8.	$\{p, \neg p, \neg s\}$	resolve 2, 3, r
9.	$\{p, \neg q, \neg s\}$	resolve 1, 8, p
10.	$\{\neg r, \neg s\}$	resolve 3, 6, p
11.	$\{\neg p, \neg r, \neg s\}$	resolve 3, 7, r
12.	$\{\neg q, \neg r, \neg s\}$	resolve 1, 11, p

As there are no further resolvents, the formula is satisfiable.

(c) *answer + explanation*

We first transform the given formula into an equivalent prenex normal form:

$$\exists x\, (\forall y\, (P(x) \to Q(y, x))) \to \forall z\, P(z)$$
$$\equiv\ \forall x\, \exists y\, \forall z\, ((P(x) \to Q(y, x)) \to P(z))$$

Next, we transform the quantifier-free part of the prenex normal form into CNF:

$$\equiv\ \forall x\, \exists y\, \forall z\, (\neg(\neg P(x) \vee Q(y, x)) \vee P(z))$$
$$\equiv\ \forall x\, \exists y\, \forall z\, ((P(x) \wedge \neg Q(y, x)) \vee P(z))$$
$$\equiv\ \forall x\, \exists y\, \forall z\, ((P(x) \vee P(z)) \wedge (\neg Q(y, x) \vee P(z))$$

We obtain an equisatisfiable Skolem normal form by replacing the existentially quantified variable y by the fresh Skolem function $f(x)$:

$$\approx\ \forall x\, \forall z\, ((P(x) \vee P(z)) \wedge (\neg Q(f(x), x) \vee P(z))$$

3 (a) *answer*

The sequent $\neg(p \land q) \vdash \neg p \lor \neg q$ is valid:

1	$\neg(p \land q)$	premise
2	$\neg(\neg p \lor \neg q)$	assumption
3	$\neg p$	assumption
4	$\neg p \lor \neg q$	$\lor$i$_1$ **??**
5	$\bot$	$\neg$e **??**, **??**
6	p	PBC **??**–**??**
7	$\neg q$	assumption
8	$\neg p \lor \neg q$	$\lor$i$_2$ **??**
9	$\bot$	$\neg$e **??**, **??**
10	q	PBC **??**–**??**
11	$p \land q$	$\land$i **??**, **??**
12	$\bot$	$\neg$e **??**, **??**
13	$\neg p \lor \neg q$	PBC **??**–**??**

(b) *answer*

The sequent $\vdash \forall x\, \exists y\, (P(x) \to Q(y)) \to \forall x\, (P(x) \to \exists y\, Q(y))$ is valid:

1		$\forall x\, \exists y\, (P(x) \to Q(y))$	assumption
2	x_0		
3		$P(x_0)$	assumption
4		$\exists y\, (P(x_0) \to Q(y))$	$\forall$e **??**
5	y_0	$P(x_0) \to Q(y_0)$	assumption
6		$Q(y_0)$	$\to$e **??**, **??**
7		$\exists y\, Q(y)$	$\exists$i **??**
8		$\exists y\, Q(y)$	$\exists$e **??**, **??**–**??**
9		$P(x_0) \to \exists y\, Q(y)$	$\to$i **??**–**??**
10		$\forall x\, (P(x) \to \exists y\, Q(y))$	$\forall$i **??**–**??**
11		$\forall x\, \exists y\, (P(x) \to Q(y)) \to \forall x\, (P(x) \to \exists y\, Q(y))$	$\to$i **??**–**??**

(c) *answer*

The sequent $\vdash \forall x\, \forall y\, (R(x, y) \to (\exists z\, (R(x, z) \wedge R(z, y))))$ is not valid. For instance, consider the model $\mathcal{M}$ consisting of the set $\{a, b\}$ with interpretation $R^{\mathcal{M}} = \{(a, b)\}$ together with $l(x) = a$ and $l(y) = b$. We have $\mathcal{M} \vDash_l R(x, y)$ but $\mathcal{M} \vDash_l \exists z\, (R(x, z) \wedge R(z, y))$ does not hold.

4 (a) *answer + explanation*

From the table

	a	$\neg a$	$\mathsf{AX}\, a$	$\mathsf{EX}\, \neg a$	$\mathsf{E}[\mathsf{AX}\, a\, \mathsf{U}\, \mathsf{EX}\, \neg a]$	φ
1	✓		✓		✓	✓
2	✓			✓	✓	✓
3	✓			✓	✓	✓
4		✓	✓		✓	✓

we conclude that the CTL formula $\varphi = \mathsf{AF}\, \mathsf{E}[\mathsf{AX}\, a\, \mathsf{U}\, \mathsf{EX}\, \neg a]$ holds in all states of $\mathcal{M}$.

(b) *answer + explanation*

For instance,

i. $\psi_1 = a \wedge \mathsf{AX}\, a$

ii. $\psi_2 = \neg\, \mathsf{EX}\, \mathsf{EX}\, \neg a$

iii. $\psi_3 = \mathsf{EX}\, \neg a \wedge \mathsf{EX}\, \mathsf{EX}\, \neg a$

iv. $\psi_4 = \neg a$

The correctness of these formulas is easily confirmed:

	a	$\neg a$	$\mathsf{AX}\, a$	$\mathsf{EX}\, \neg a$	$\mathsf{EX}\, \mathsf{EX}\, \neg a$	ψ_1	ψ_2	ψ_3	ψ_4
1	✓		✓		✓	✓			
2	✓			✓			✓		
3	✓			✓	✓			✓	
4		✓	✓		✓				✓

(c) *answer + explanation*

For instance, $\chi = \mathsf{X}\, a$. We have $\mathcal{M}, 2 \not\models \chi$ because the path $2\,4\,3^\omega$ does not satisfy χ. Also, $\mathcal{M}, 2 \not\models \neg\chi$ because the path $2\,1\,3^\omega$ satisfies χ.

5	true	false	statement
	X		The set $\{\mathsf{EX}, \mathsf{EU}, \mathsf{AF}\}$ is adequate for CTL.
	X		The formulas $(p \lor q) \land \neg p$ and $\top$ are equisatisfiable.
		X	Resolution is sound and complete for predicate logic.
		X	Intuitionistic logicans do not use LEM, PBC and $\rightarrow$e.
	X		Deciding the satisfiability of CNF formulas is NP-complete.
	X		The formula $(p \land q \rightarrow s) \land (s \rightarrow r) \land (q \rightarrow \bot)$ is a Horn formula.
		X	Every boolean function has a unique representation as reduced BDD.
		X	The set $[\![\mathsf{AF}\,\varphi]\!]$ is the least fixed point of function $F_{\mathsf{AF}}(X) = [\![\varphi]\!] \cap \mathsf{pre}_{\forall}(X)$.
	X		The sequent $\exists x\, \exists y\, (P(x,y) \lor P(y,x)), \neg\exists x\, P(x,x) \vdash \exists x\, \exists y\, \neg(x = y)$ is valid.
		X	An n-ary boolean function f is not self-dual if and only if $f(b_1, \ldots, b_n) = f(\overline{b_1}, \ldots, \overline{b_n})$ for all $b_1, \ldots, b_n \in \{0, 1\}$.