## Exercises Week 12

Consider the following simplification rules for type inference problems.

$$
\begin{array}{lll}
A \rhd z : \tau & \rightsquigarrow \bot & \text{if } z \notin \mathrm{dom}(A) \\
A \rhd z : \tau & \rightsquigarrow A(z) \doteq \tau & \text{if } z \in \mathrm{dom}(A) \\
A \rhd (\texttt{fun}\ x \to e) : \tau & \rightsquigarrow \exists \alpha_1, \alpha_2.(A, x : \alpha_1 \rhd e : \alpha_2 \ \wedge\ \tau \doteq \alpha_1 \to \alpha_2) & \\
A \rhd e_1 e_2 : \tau & \rightsquigarrow \exists \alpha.(A \rhd e_1 : \alpha \to \tau \ \wedge\ A \rhd e_2 : \alpha) & \\
A \rhd \texttt{let}\ x = e_1\ \texttt{in}\ e_2 : \tau & \rightsquigarrow \exists \alpha.(A \rhd e_1 : \alpha \ \wedge\ A, x : \alpha \rhd e_2 : \tau) &
\end{array}
$$

1. Rewrite to unification problems.
   (a) $A \rhd (\texttt{fun}\ x \to x)\ \texttt{true} : \alpha$
   (b) $A \rhd (\texttt{fun}\ x \to x + x) : \alpha$
   (c) $A \rhd (\texttt{fun}\ x \to x) : \alpha$

   Here $A$ is the type environment defined by $A(\texttt{true}) = \texttt{bool}$ and $A(+) = \texttt{int} \to \texttt{int} \to \texttt{int}$.

2. Solve the unification problems.