Schedule

SCHEDULE

w	date	topic
~~7~~	~~November 25~~	~~introduction~~
8	December 2	higher-order functions, lists, trees
9	December 9	graphs, combinatorics
10	December 16	program reasoning
11	January 13	λ and interpreter
12	January 20	type system
13	January 27	exam part 2

CONTENTS

1. higher-order functions on lists
2. exceptions
3. algebraic data types
4. homework 2

Higher-order Functions

Higher-Order and Anonymous Functions, Map

- (higher-order) function can take functions and return function
- anonymous function is constructed by fun

DEFINITION

$$\text{List.map } f\ [x_1; \cdots; x_n] = [f\ x_1; \cdots; f\ x_n]$$

EXAMPLE

```
# let plus1 x = x + 1;;
# List.map plus1 [1; 2; 3];;
- : int list = [2; 3; 4]
# List.map (fun x -> x + 1) [1; 2; 3];;
- : int list = [2; 3; 4]
```


Fold

DEFINITION

$$\begin{aligned} \text{List.fold\_left } (\circ)\ e\ [x_1; \cdots; x_n] &= (e \circ x_1) \circ \cdots \circ x_n \\ \text{List.fold\_right } (\circ)\ [x_1; \cdots; x_n]\ e &= x_1 \circ \cdots \circ (x_n \circ e) \end{aligned}$$

EXAMPLE

```
# List.fold_left (+) 0 [1;2;3];;
- : int = 6
# List.fold_left (fun x y -> x - y) 10 [1;2;3];;
- : int = 4
# List.fold_right (@) [[1;2;3]; [4;5]; [6]] [];;
- : int list = [1; 2; 3; 4; 5; 6]
```

EXERCISES

- `sum : int list -> int`
- `concat : 'a list list -> 'a list`

Filter

$$\begin{aligned}
\text{List.filter even} \quad & [\,] &= & \; [\,] \\
\text{List.filter even} \; (& 4 :: [\,]) &= & \; 4 :: [\,] \\
\text{List.filter even} \; (& 3 :: 4 :: [\,]) &= & \; 4 :: [\,] \\
\text{List.filter even} \; (& 2 :: 3 :: 4 :: [\,]) &= & \; 2 :: 4 :: [\,] \\
\text{List.filter even} \; (& 1 :: 2 :: 3 :: 4 :: [\,]) &= & \; 2 :: 4 :: [\,]
\end{aligned}$$

where even $x \; = \; x$ mod $2 = 0$

```
let rec filter p = function
  | [] ->
  | x :: xs when p x ->
  | _ :: xs ->
```


Partition

$$\begin{aligned}
\text{partition even} \quad & [\,] &= & \; ([\,], [\,]) \\
\text{partition even} \; (& 3 :: [\,]) &= & \; ([\,], 3 :: [\,]) \\
\text{partition even} \; (& 2 :: 3 :: [\,]) &= & \; (2 :: [\,], 3 :: [\,]) \\
\text{partition even} \; (& 1 :: 2 :: 3 :: [\,]) &= & \; (2 :: [\,], 1 :: 3 :: [\,])
\end{aligned}$$

```
let rec partition p = function
  | [] ->
  | x :: xs ->
      let ys, zs = partition p xs in
```

Quicksort

Hoare 1960

$$\text{qsort } [\,] = [\,]$$
$$\text{qsort } (x :: xs) = \text{qsort } [y \mid y \leftarrow xs, y \leqslant x] \mathbin{@} [x] \mathbin{@} \text{qsort } [y \mid y \leftarrow xs, x < y]$$

- based on divide and conquer
- simpler than definition of sorted list

EXERCISE

implement it!

Divide and Conquer

$$\text{qsort } [3;5;2;4;1]$$
$$=$$
$$\text{qsort } [2;1] \mathbin{@} [3] \mathbin{@} \text{qsort } [5;4]$$
$$=$$
$$(\text{qsort } [1] \mathbin{@} [2] \mathbin{@} \text{qsort } [\,]) \mathbin{@} [3] \mathbin{@} (\text{qsort } [4] \mathbin{@} [5] \mathbin{@} \text{qsort } [\,])$$
$$=$$
$$(([\,] \mathbin{@} [1] \mathbin{@} [\,]) \mathbin{@} [2] \mathbin{@} [\,]) \mathbin{@} [3] \mathbin{@} (([\,] \mathbin{@} [4] \mathbin{@} [\,]) \mathbin{@} [5] \mathbin{@} [\,])$$
$$=$$
$$([1] \mathbin{@} [2] \mathbin{@} [\,]) \mathbin{@} [3] \mathbin{@} ([4] \mathbin{@} [5] \mathbin{@} [\,])$$
$$=$$
$$[1;2] \mathbin{@} [3] \mathbin{@} [4;5]$$
$$=$$
$$[1;2;3;4;5]$$

Exceptions

Raising Exceptions

- `exception` declares exception
- `raise` raises exception

EXAMPLE

$$\text{assoc } ''x'' \ [(''x'', 1); (''y'', 2)] = 1$$
$$\text{assoc } ''y'' \ [(''x'', 1); (''y'', 2)] = 2$$
$$\text{assoc } ''z'' \ [(''x'', 1); (''y'', 2)] = \text{raise Not\_found}$$

```
exception Not_found
let rec assoc x = function
  | [] ->
  | (y, z) :: _ when x = y ->
  | _ :: alist ->
```

Catching Exceptions

- exception slips through functions
- `try ... with ...` pattern-matches exception to catch

EXAMPLE

$$1 + \text{raise Not\_found} = \text{raise Not\_found}$$
$$\text{try } 1 + \text{raise Not\_found with Not\_found} \rightarrow 0 = 0$$
$$\text{try } 1 + 2 \text{ with Not\_found} \rightarrow 0 = 3$$

Algebraic Data Types

Algebraic Data Types

- algebraic data type is declared by type
- consists of values and data constructors

EXAMPLE

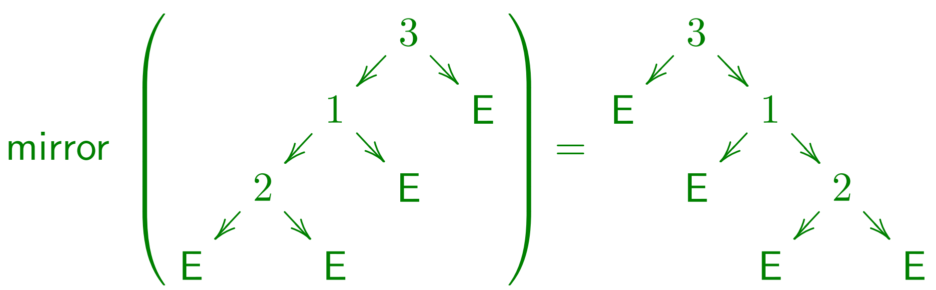
```
type 'a tree = Empty | Node of 'a tree * 'a * 'a tree
```

$$\mathsf{Empty}$$
$$\mathsf{Node}(\mathsf{Empty}, 2, \mathsf{Empty})$$
$$\mathsf{Node}(\mathsf{Node}(\mathsf{Empty}, 2, \mathsf{Empty}), 1, \mathsf{Empty})$$

Pattern Matching

EXAMPLE

$$\begin{aligned} & \mathsf{mirror}\ \ (\mathsf{Node}(\mathsf{Node}(\mathsf{Node}(\mathsf{Empty}, 2, \mathsf{Empty}), 1, \mathsf{Empty}), 3, \mathsf{Empty})) \\ = \ & \mathsf{Node}(\mathsf{Empty}, 3, \mathsf{Node}(\mathsf{Empty}, 1, \mathsf{Node}(\mathsf{Empty}, 2, \mathsf{Empty}))) \end{aligned}$$




```
let rec mirror = function
  | Empty ->
  | Node (l, x, r) ->
```

EXERCISES

- `preorder : 'a tree -> 'a list`
- `postorder : 'a tree -> 'a list`

Expressions and Evaluation

EXAMPLE

```
type expr =
  | Const of int
  | Add of expr * expr
  | Mul of expr * expr
```

$$\begin{aligned}
\mathsf{eval}(\mathsf{Const}\ 2) &= 2 \\
\mathsf{eval}(\mathsf{Mul}(\mathsf{Const}\ 2, \mathsf{Const}\ 3)) &= 6 \\
\mathsf{eval}(\mathsf{Add}(\mathsf{Const}\ 1, \mathsf{Mul}(\mathsf{Const}\ 2, \mathsf{Const}\ 3))) &= 7
\end{aligned}$$

```
let rec eval = function
  | Const n ->
  | Add (e1, e2) ->
  | Mul (e1, e2) ->
```

Printing Data Structures

- `open` in OCaml ≈ `import` in Java
- module `Format` includes `fprintf`, `std_formatter`, ...

```
# open Format;;

# let rec fprint_expr formatter = function
  | Const n -> fprintf formatter "%d" n
  | Add (e1, e2) ->
      fprintf formatter "(%a + %a)"
        fprint_expr e1 fprint_expr e2
  | Mul (e1, e2) ->
      fprintf formatter "(%a * %a)"
        fprint_expr e1 fprint_expr e2;;

# fprint_expr std_formatter
    (Add (Const 1, Mul (Const 2, Const 3)));;
(1 + (2 * 3))- : unit
```


Installing Printer

```
# #install_printer fprint_expr;;
# Mul (Const 2, Add (Const 3, Const 4));;
- : expr = (2 * (3 + 4))

# #trace eval;;
eval is now traced.
# eval (Mul (Const 2, Add (Const 3, Const 4)));;
eval <-- (2 * (3 + 4))
eval <-- (3 + 4)
eval <-- 4
eval --> 4
eval <-- 3
eval --> 3
eval --> 7
eval <-- 2
eval --> 2
eval --> 14
- : int = 14
```

Homework: Stack-based Virtual Machines

Stack-based VM

- instruction set

```
type instr = Push | Addint | Mulint | Value of int
```

- bytecode compiler

```
# compile (Mul (Const 10, Add (Const 20, Const 30)));;
- : instr list = [Push; Value 10; Push; Value 20;
                  Push; Value 30; Addint; Mulint]
```

- bytecode interpreter

```
# interpret [Push; Value 10; Push; Value 20;
             Push; Value 30; Addint; Mulint]
- : int = 500
```

REMARK

use 16/32/64/128-bit integers for instruction in practice

Interpreting Bytecode

$$\begin{array}{l} \qquad\quad 0 \quad\; 1 \qquad\quad\;\; 2 \quad\; 3 \qquad\quad\;\; 4 \quad\; 5 \qquad\quad\;\; 6 \qquad\; 7 \\ \mathsf{code} = [\ \mathsf{Push};\ \mathsf{Value}\ 10;\ \mathsf{Push};\ \mathsf{Value}\ 20;\ \mathsf{Push};\ \mathsf{Value}\ 30;\ \mathsf{Addint};\ \mathsf{Mulint}\ ] \end{array}$$

$$\begin{array}{rcll} & & \qquad\qquad\qquad\quad \mathsf{pc} & \qquad\qquad \mathsf{stack} \\ \mathsf{interpret\ code} & = & \mathsf{interpret'\ code}\ (0, & \qquad\qquad\qquad\quad [\,]) \\ & = & \mathsf{interpret'\ code}\ (2, & \qquad\qquad 10 :: [\,]) \\ & = & \mathsf{interpret'\ code}\ (4, & \qquad 20 :: 10 :: [\,]) \\ & = & \mathsf{interpret'\ code}\ (6, & 30 :: 20 :: 10 :: [\,]) \\ & = & \mathsf{interpret'\ code}\ (7, & \qquad 50 :: 10 :: [\,]) \\ & = & \mathsf{interpret'\ code}\ (8, & \qquad\qquad 500 :: [\,]) \\ & = & 500 & \end{array}$$

Compiling Expressions

$$\begin{array}{ll} & \mathsf{compile}\ (\mathsf{Mul}\ (\mathsf{Const}\ 10, \mathsf{Add}\ (\mathsf{Const}\ 20, \mathsf{Const}30))) \\ = & \mathsf{compile}\ (\mathsf{Const}\ 10)\ @ \\ & \mathsf{compile}\ (\mathsf{Add}\ (\mathsf{Const}\ 20, \mathsf{Const}\ 30))\ @ \\ & [\mathsf{Mulint}] \\ = & [\mathsf{Push}; \mathsf{Value}\ 10]\ @ \\ & \mathsf{compile}\ (\mathsf{Add}\ (\mathsf{Const}\ 20, \mathsf{Const}\ 30))\ @ \\ & [\mathsf{Mulint}] \\ = & \cdots \\ = & [\mathsf{Push}; \mathsf{Value}\ 10; \\ & \ \mathsf{Push}; \mathsf{Value}\ 20; \\ & \ \mathsf{Push}; \mathsf{Value}\ 30; \\ & \ \mathsf{Addint}; \\ & \ \mathsf{Mulint}] \end{array}$$