

**Exercises.**

10.0 Study Chapter 5.9, 6.1–6.4

10.1 Exercise 5.9.2

10.2 Exercise 6.1.1

10.3 Exercise 6.2.1

**Optional Exercises.**

1. Let  $\mathbf{G} = (\mathbf{V}, \mathbf{E})$  be a finite directed graph. We write  $\text{path}(v, w)$  to indicate that there exists a path from  $v$  to  $w$  in  $\mathbf{G}$ . Then no first-order formulas  $X(x, y)$  can exist, such that  $X(x, y)$  is true in  $\mathbf{G}$  for some assignment  $\mathbf{A}$  iff  $\text{path}(x^{\mathbf{A}}, y^{\mathbf{A}})$  holds. I.e. reachability is not first-order definable.
2. Let  $\mathbf{G} = (\mathbf{V}, \mathbf{E})$  be defined as above. There exists a second-order formula  $\text{path}(P)$ , expressing that  $P$  is path in  $\mathbf{G}$ .