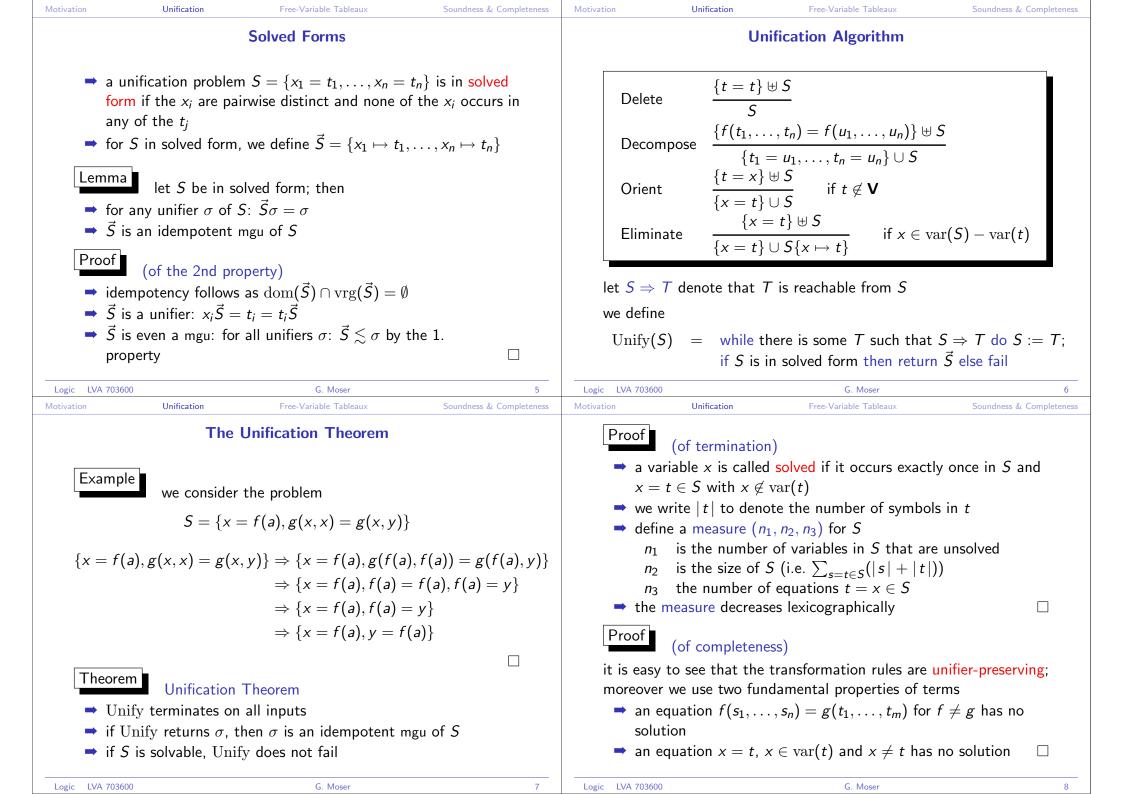
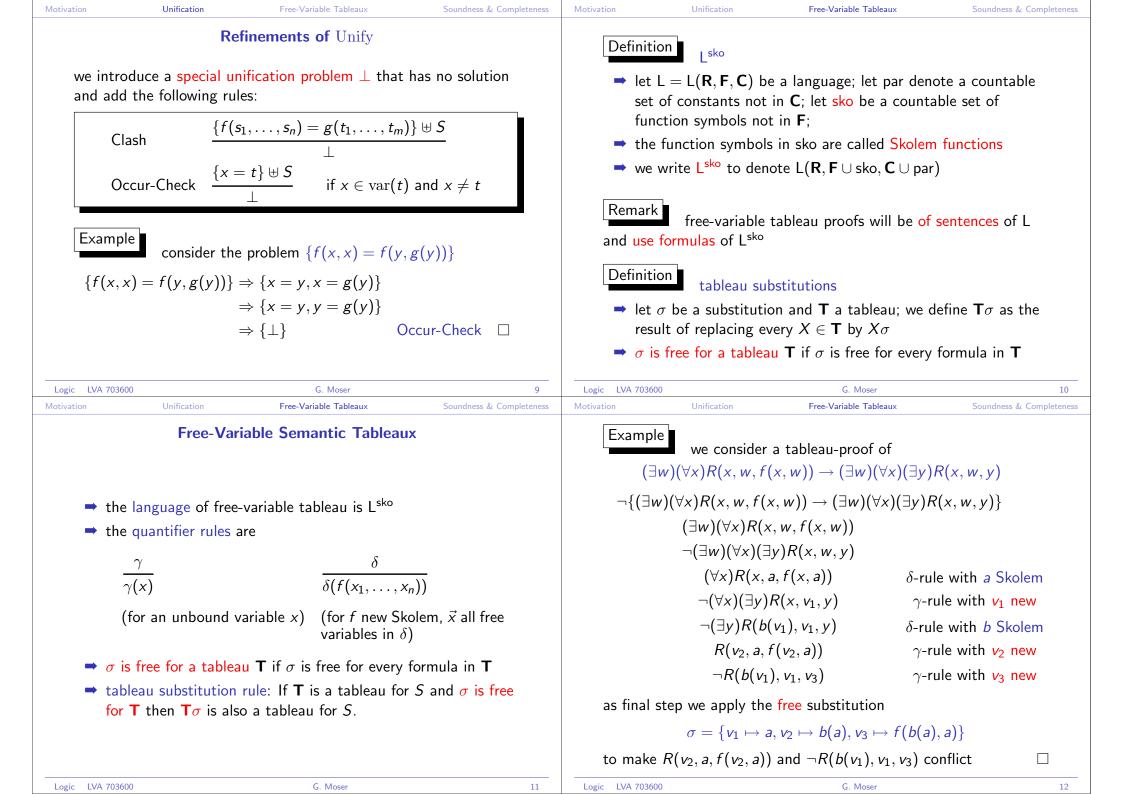
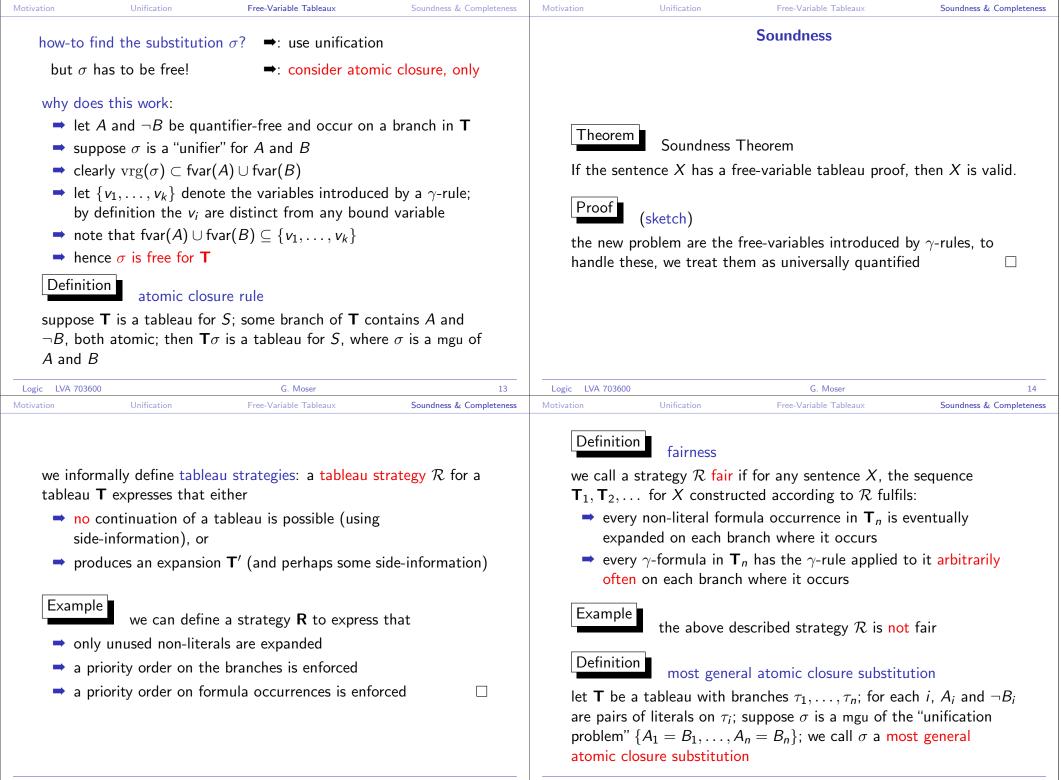
Motivation Unification Free-Variable Tableaux Soundness & Completeness	Motivation Unification Free-Variable Tableaux Soundness & Completeness			
	Motivation			
	let us look at a tableau proof of $\{(orall x)(P(x) \lor Q(x)), (orall x) \lnot P(x), (\exists x) \lnot Q(x)\}$			
Logic LVA 703600 VU3				
http://cl-informatik.uibk.ac.at/teaching/ws05/logic/ Georg Moser (VU) ¹ Christian Vogt (VU) ² ¹ georg.moser@uibk.ac.at office hours: Thursday 1pm-3pm ² christian.vogt@uibk.ac.at office hours: Tuesday 9am-11am Autumn 2005	$\begin{array}{c c} closed & free-variable & \underline{\gamma}\\ (\forall x)(P(x) \lor Q(x)) & (\forall x)(P(x) \lor Q(x)) & \overline{\gamma(x)}\\ (\forall x)\neg P(x) & (\forall x)\neg P(x) & (for an \\ unbound \\ (\exists x)\neg Q(x) & (\exists x)\neg Q(x) & variable x)\\ \neg Q(c) & \neg Q(c) \\ \neg P(d) & \neg P(x) & \underline{\delta}\\ P(c) \lor Q(c) & P(y) \lor Q(y) & (for f new, \vec{x} \\ P(c) & Q(c) & P(y) \lor Q(y) & all free \\ \neg P(c) & \neg P(c) & variables in \delta) \end{array}$			
Logic LVA 703600 G. Moser 1 Motivation Unification Free-Variable Tableaux Soundness & Completeness Unification	Logic LVA 703600 G. Moser 2 Motivation Unification Free-Variable Tableaux Soundness & Completeness Example			
to close the tableau, we have to find σ such that $Q(c)\sigma = Q(y)\sigma$ $P(x)\sigma = P(y)\sigma$ obviously $\sigma = \{x \mapsto c, y \mapsto c\}$ would be sufficient \Rightarrow a unification problem is a finite set of equations $S = \{s_1 = {}^{?} t_1, \dots, s_n = {}^{?} t_n\}$	 the unification problem {f(y, h(a)) = f(h(x), h(z))} is solvable with σ₁ = {y ↦ h(x), z ↦ a} σ₂ = {x ↦ k(w), y ↦ h(k(w)), z ↦ a} but σ₁ ≤ σ₂ and σ₁ is a mgu the unification problem {f(x, x) = f(a, b)} is not solvable 			
 a unifier of S is a substitution such that s_iσ = t_iσ for all i = 1,, n a substitution σ is more general than a substitution τ, if τ = σρ for some substitution ρ; we write σ ≤ τ a most general unifier (mgu) is a unifier σ s.t. for all unifiers τ: σ ≤ τ 	Lemma idempotent substitutions a substitution σ is idempotent if $\sigma = \sigma\sigma$; then \Rightarrow a substitution σ is idempotent iff $dom(\sigma) \cap vrg(\sigma) = \emptyset$ Theorem If a unification problem S is solvable, then it has an idempotent mgu.			

3

Logic LVA 703600







15 Logic LVA 703600

16

Motiva	tion	Unification	Free-Variable Tableaux	Soundness & Completeness	Motivation	Unification	Free-Variable Tableaux	Soundness & Completeness
Theorem Completeness Let \mathcal{R} be any fair tableau strategy. If X is a valid sentence of L, X has a tableau proof which fulfils: • all tableau expansion rules applications come first and are according to rule \mathcal{R} • a single tableau substitution rule follows, using a substitution σ that is a most general atomic closure substitution				→ un → un → fre → rei → ta	ification	Summary by transformation c tableaux variable tableaux mess	Soundness & Completeness	
Logi	c LVA 703600		G. Moser	17	Logic LVA 703	600	G. Moser	18