University of Innsbruck
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WS 2006/07
February 1, 2007
Final Exam
Time: 90 minutes

Automated Theorem Proving in Isabelle/HOL
LVA 703861

1 Type Inference in Simply-Typed Lambda Calculus **10 points**

Give a most general type for the following λ-term. Show the full typing derivation.

$$\lambda x\, y.\, x\, (x\, y)$$

2 Natural Deduction **10 points**

Give a complete set of natural-deduction style introduction and elimination rules (without using conjunction/disjunction) for *exclusive disjunction* (xor, $\oplus$), defined by the following truth table:

P	Q	$P \oplus Q$
False	False	False
False	True	True
True	False	True
True	True	False

3 Modeling/Formalization **10 points**

Give a formalization of the following puzzle in (a suitable subset of) higher-order logic. Explain your formalization. (You do *not* need to solve the puzzle.)

> Human observers in this exclusive club on Ganymede can't distinguish Martians from Venusians, males from females, except for the fact that Venusian women and Martian men always tell the truth and Venusian men and Martian women always lie. (Everyone is either Martian or Venusian, and either male or female.) A says "B is from Venus." B says "A is from Mars." A says "B is male." B says "A is female." Who's what (sex and planet of origin)?

4 Inductive Definitions **15 points**

Give a formalization of the following puzzle in (a suitable subset of) higher-order logic. Explain your formalization. (You do *not* need to solve the puzzle.)

> There are 3 missionaries, 3 cannibals, and a boat on the west bank of a river. All wish to cross, but the boat holds at most 2 people. If the cannibals ever outnumber the missionaries on either bank of the river the outnumbered missionaries will be eaten. Can they all safely cross the river? (The boat cannot cross empty.)

Hint: Find a suitable notion of "state". Characterize the reachable states inductively.

5 Isabelle/Isar **15 points**

Give an Isar proof of the following lemma. The general proof structure should resemble that of a detailed informal (not necessarily natural deduction) proof.

lemma "($\exists x.\ \forall y.\ P\ x\ y$) $\Longrightarrow$ ($\forall y.\ \exists x.\ P\ x\ y$)"

You know my methods. Apply them!
Sherlock Holmes